



# Energies of Doubly Excited ${}^1,{}^3P^\circ$ Resonances in He-like Systems Below the N = 2–14 Hydrogenic Threshold

Jean Kouhissore Badiane<sup>1,\*</sup>, Mamadou Dioulde Ba<sup>1</sup>, Abdourahmane Diallo<sup>2</sup>, Momar Talla Gning<sup>2</sup>

<sup>1</sup>Department of Physics, UFR Sciences and Technologies, University Assane Seck of Ziguinchor, Ziguinchor, Senegal

<sup>2</sup>Department of Physics Chemistry, UFR Sciences and Technologies, University Ibader Thiam of Thies, Thies, Senegal

**Email address:**

j.badiane287@zig.univ.sn (J. K. Badiane), m.ba2664@zig.univ.sn (M. D. Ba), a.diallo1483@zig.univ.sn (A. Diallo), m.gning20150917@zig.univ.sn (M. T. Gning)

\*Corresponding author

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**Abstract:** In this present work, resonance energies and excitation energies of doubly  ${}^1,{}^3P^\circ$  excited states of the helium isoelectronic sequence are calculated. The doubly excited states investigated are labelled in the (K, T, A) classification scheme. The energies are calculated in the framework of the variationnal procedure of the Screening constant by unit nuclear charge (SCUNC) formalism. The results obtained compared very well with theoretical and experimental literature values. The possibility to use the SCUNC formalism report rapidly with an excellent accuracy the position of the excitation resonances as well as their width within simple analytical formulae is demonstrated. It is demonstrated that the SCUNC-method can be used to assist fruitfully experiments for identifying narrow resonance energies. Thus, our results can be used as reference data for the interpretation of atomic spectra for the diagnosis of astrophysical and laboratory plasma. In the present work, a new correlated wave function is presented to express analytically the resonance energies and excitation energies of doubly  ${}^1,{}^3P^\circ$  excited states in the He-like systems.

**Keywords:** Doubly Excited  ${}^1,{}^3P^\circ$  Resonances, Semi-empirical Calculations, Screening Constant by Unit Nuclear Charge, Rydberg Series, He-isoelectronic Sequence, (K, T, A) Classification Scheme

## 1. Introduction

Over the last two decades, various methods have been performed to understand electron-electron correlation effects in doubly excited states of He-like systems. Although many accurate data have been tabulated for these doubly excited states, the methods used require in general, complexity in the variationnal procedure along with the use of computational codes. Subsequently, energy of the doubly excited states of He-like systems cannot be expressed analytically from various *ab initio* methods such as the complex-coordinate rotation method [3, 4], the variationnal method of Hylleraas [6], the double sums over the total hydrogen spectrum formalism [7], the formalism of the Feshbach projection operators [8], the saddle-point complex-rotation method with B-spline functions [9], the truncated diagonalization method [10], to name a few. In contrast with these *ab initio* methods

applied in the analysis of two atomic systems spectra, the Screening constant by unit nuclear charge (SCUNC) formalism has the advantages to provide accurate energy position within a simple semi-empirical procedure as well as for the doubly ( $Nlnl'; {}^{2S+1}L^\pi$ ) excited state [12, 13] than in the mixed configurations based on the classification scheme designated by the notation  ${}_N(K, T)_n^A {}^{2S+1}L^\pi$  [15, 16]. However, it is also challenging to succeed on interpreting quantitatively the physical meaning of the K correlation quantum number. In fact, if T is roughly speaking the projection of L onto the interelectronic axis and describes then the orientations between the orbitals of the two electrons, K is related to the cosinus of the interelectronic angle  $\theta_{12}$  as  $K \approx -\langle r_< \cos \theta_{12} \rangle$  where  $r_<$  denotes the radius of the inner electron. Physically, the larger the positive value of K, the value of  $-\cos \theta_{12}$  is closer to unity. So, the variationnal procedure of the SCUNC formalism has been applied

successfully for the ground state [11] and doubly excited ( $nl$ )<sup>2</sup> states [17] of He-like ions. The semi-empirical procedure of the same formalism is based on experimental data. But this semi-empirical procedure requires the use of experimental data as input to evaluate the fitting parameters in the energy expressions. This means that, when experimental data are unavailable, the SCUNC formalism becomes unsuitable. As most of the doubly excited states in He-like ions no experimental data are available, it becomes very challenging to develop the variationnal procedure of the SCUNC formalism on these excited states. Then, the goal of the present work is to apply for the first time, the variationnal procedure of the SCUNC formalism in the calculation of energies of the doubly  ${}_{\text{N}}(\text{K}, \text{T})_n^A {}^{2S+1}L^\pi$  excited. The present study is focused firstly on the doubly  ${}_{\text{N}}(\text{K}, \text{T})_n^A {}^{1,3}\text{P}^0$  excited

$$E({}_{\text{N}}(\text{K}, \text{T})_n^A {}^{2S+1}L^\pi) = -Z^2 \left\{ \frac{1}{N^2} + \frac{1}{n^2} \left( 1 - \beta(Nlnl'; {}^{2S+1}L^\pi; Z) \right)^2 \right\} \text{Ryb} \quad (1)$$

In this equation, the principal quantum numbers  $N$  and  $n$ , are respectively for the inner and the outer electron of He-isoelectronic series. In this equation, the  $\beta$ -parameters are screening constant by unit nuclear charge expanded in inverse powers of  $Z$  and given by

$$\beta(Nlnl'; {}^{2S+1}L^\pi; Z) = \sum_{k=1}^q f_k \left( \frac{1}{Z} \right)^k \quad (2)$$

Where  $f_k = f_k(Nlnl'; {}^{2S+1}L^\pi)$  are parameters to be evaluated. With respect to the new classification scheme, eq. (1) takes the form

$$E({}_{\text{N}}(\text{K}, \text{T})_n^A {}^{2S+1}L^\pi) = -Z^2 \left\{ \frac{1}{N^2} + \frac{1}{n^2} \left( 1 - \beta[{}_{\text{N}}(\text{K}, \text{T})_n^A {}^{2S+1}L^\pi; Z] \right)^2 \right\} \text{Ryb} \quad (3)$$

Using eq. (2), we get from eq. (3)

$$E({}_{\text{N}}(\text{K}, \text{T})_n^A {}^{2S+1}L^\pi) = -Z^2 \left\{ \frac{1}{N^2} + \frac{1}{n^2} \left( 1 - \sum_{k=1}^q f_k({}_{\text{N}}(\text{K}, \text{T})_n^A {}^{2S+1}L^\pi) \left( \frac{1}{Z} \right)^k \right)^2 \right\} \text{Ryb} \quad (4)$$

For all the doubly excited states investigated in the present work and lying to the  $\text{N} = 2$ –14 hydrogenic threshold, resonance energy is expressed as follows using eq. (4)

$$E({}_{\text{N}}(\text{K}, \text{T})_n^A {}^{2S+1}L^\pi) = -Z^2 \left\{ \frac{1}{N^2} + \frac{1}{n^2} \left( 1 - \frac{f_1[(K, T, A) {}^{2S+1}L^\pi]}{Z(n-1)} - \frac{f_2[(K, T, A) {}^{2S+1}L^\pi]}{Z} \right)^2 \right\} \text{Ryb} \quad (5)$$

The  $f_1[(K, T, A) {}^{2S+1}L^\pi]$  and  $f_2[(K, T, A) {}^{2S+1}L^\pi]$  screening constants in eq. (5) are expressed in term of  $f_0$  who denote the value of the screening constant of the  $\alpha_0$ -variational parameter as follows:

$$f_1[(K, T, A) {}^{2S+1}L^\pi] = \frac{(n-K-S)(S+T)-s}{(S+T)} \times f_0 \quad (6)$$

$$f_2[(K, T, A) {}^{2S+1}L^\pi] = f_0$$

In this equation,  $K$ ,  $T$ , and  $A$  are correlation quantum numbers.  $S$  is the total spin and  $s$  is the electron spin. Using eq. (6), we get from eq. (5)

$$E = -Z^2 \left\{ \frac{1}{N^2} + \frac{1}{n^2} \left( 1 - \frac{(n-K-S)(S+T)-s}{Z(n-1)(S+T)} \times f_0 - \frac{1}{Z} \times f_0 \right)^2 \right\} \text{Ryb} \quad (7)$$

For all states studied ( $T=1$ ), the simplified expression of resonance energies is given by:

$$E = -Z^2 \left\{ \frac{1}{N^2} + \frac{1}{n^2} \left( 1 - \frac{(n-K-S)(S+1)-s}{Z(n-1)(S+1)} \times f_0 - \frac{1}{Z} \times f_0 \right)^2 \right\} \text{Ryb} \quad (8)$$

## 2.2. Appropriated Correlated Wave Function

To evaluated theoretically the  $f_0$ -screening constant in eq. (8), the first step consists of constructing the appropriated

states of two-electron atomic systems. After, a brief description of the variationnal procedure of the SCUNC formalism adopted in this work will be done. Next, the results obtained are displayed and discussed. And finally a conclusion will be made.

## 2. Theory

### 2.1. General Formalism of the Method

In the framework of the Screening Constant by Unit Nuclear Charge formalism, the total energy of the doubly ( $Nlnl'; {}^{2S+1}L^\pi$ ) excited states is expressed in the form (in Rydberg)

correlated wave function for the doubly  $Nsnp$   $^{1,3}P^{\circ}$  excited states. After, the  $f_0$ -parameter is determined from the Ritz's variation principle.

For the helium isoelectronic sequence containing two electrons,  $r_1$  and  $r_2$  being radial coordinates, wave function expression becomes in the framework of the independent particles model

$$\Psi_{Nlnl'}(r_1, r_2) = r_1^l r_2^{l'} \times \exp\left(-\frac{z}{a_0}(r_1 + r_2)\right) \quad (9)$$

Taking into account electron-electron correlation effects occurring in He-like systems, the  $Z^*$ - effective charge number is introduced by constructing the correlated wave function as follows:

$$\Psi_{Nlnl'}(r_1, r_2) = r_1^l r_2^{l'} \times \exp\left(-\frac{Z^*}{N \times n \times a_0}(nr_1 + Nr_2)\right) \quad (10)$$

where the effective charge number is given by

$$Z^* = \frac{n \times ((2L+2) \times (n+N) - L)}{(2L+2) \times (n+N)} Z \left(1 - \frac{f_0}{Z}\right) \quad (11)$$

As far as the variationnal parameter  $\alpha$  of the lower energy is concerned, it is given by

$$\alpha = \frac{Z^*}{n} = \frac{(2L+2) \times (n+N) - L}{(2L+2) \times (n+N)} Z \left(1 - f_0 \times \frac{1}{Z}\right) \quad (12)$$

In atomic units,  $a_0 = 1$ . Taking into account eq. (10), the correlated wave function is written in the form:

$$\Psi_{Nlnl'}(r_1, r_2) = r_1^l r_2^{l'} \times \exp\left(-\frac{\alpha}{N}(nr_1 + Nr_2)\right) \quad (13)$$

For  $Nsnp$  states ( $l = 0, l' = 1$ ), the correlated wave function eq. (13) becomes

$$\Psi_{Nsnp}(r_1, r_2) = r_2 \times \exp\left(-\frac{\alpha}{N}(nr_1 + Nr_2)\right) \quad (14)$$

However, the Hamiltonian of the helium isoelectronic series is given by (in atomic units)

$$H = -\frac{1}{2}\Delta_1 - \frac{1}{2}\Delta_2 - \frac{Z}{r_1} - \frac{Z}{r_2} + \frac{1}{r_{12}} \quad (15)$$

In the framework of the Ritz' variation principle, the energy  $E(\alpha)$  is obtained from the condition

$$\left(\frac{d\langle H \rangle(\alpha)}{d\alpha}\right)_{\alpha=\alpha_0} = 0 \quad (16)$$

In eq. (2.16),  $E(\alpha) = \langle H \rangle(\alpha)$  with

$$E(\alpha) = \langle H \rangle(\alpha) = \frac{\langle \Psi(\alpha) | H | \Psi(\alpha) \rangle}{\langle \Psi(\alpha) | \Psi(\alpha) \rangle} \quad (17)$$

The integration of eq. (17) is facilitated by turning into account a change of variables using elliptical coordinates [14]

$$s = (r_1 + r_2); t = (r_1 - r_2); u = r_{12} \quad (18)$$

$$d\tau = d^3 r_1 d^3 r_2 = 2\pi^2 (s^2 - t^2) u \, ds \, dt \, du.$$

Using these elliptical coordinates, eq. (17) is rewritten as follows

$$NE(\alpha) = \int_0^\infty ds \int_0^s du \int_0^u dt \left\{ u(s^2 - t^2) \times \left[ \left(\frac{\partial \Psi}{\partial s}\right)^2 + \left(\frac{\partial \Psi}{\partial t}\right)^2 + \left(\frac{\partial \Psi}{\partial u}\right)^2 \right] + 2 \left(\frac{\partial \Psi}{\partial u}\right) \times \left[ s(u^2 - t^2) \frac{\partial \Psi}{\partial s} + t(s^2 - u^2) \frac{\partial \Psi}{\partial t} \right] - \Psi^2 \times [4Zsu - s^2 + t^2] \right\} \quad (19)$$

As far as the correlated wave function is concerned, it is expressed in the form

$$\Psi_{Nsnp}(s, t, u) = \left(\frac{s-t}{2}\right) \times \exp\left(-\frac{\alpha}{2N}[(n+N)s + (n-N)t]\right) \quad (20)$$

### 3. Results and Discussion

Using the correlated wave function eq. (20), eq. (19) provides

$$E(\alpha) = \frac{1403}{2538} \alpha^2 - \left( \frac{295}{282} Z - \frac{20}{47} \right) \alpha \quad (21)$$

From the variationnal principle, one can write

$$\left( \frac{dE(\alpha)}{d\alpha} \right)_{\alpha=\alpha_0} = 0 \quad (22)$$

So, eq. (21) provides

$$\alpha_0 = \frac{19}{20} \left( Z - \frac{24}{59} \right) = \frac{19}{20} Z \left( 1 - \frac{24}{59} \times \frac{1}{Z} \right) \quad (23)$$

Eq. (23) is deduced from general eq. (12) reminded below

$$\alpha = \frac{z^*}{n} = \frac{(2L+2) \times (n+N) - L}{(2L+2) \times (n+N)} Z \left( 1 - f_0 \times \frac{1}{Z} \right) \quad (24)$$

for the Nsnp states

$$\alpha_0 = \frac{19}{20} Z \left( 1 - \frac{24}{59} \times \frac{1}{Z} \right) = \frac{19}{20} Z \left( 1 - f_0 \times \frac{1}{Z} \right) \quad (25)$$

Let us use approximately

$$f_0 = \frac{24}{59} = 0,40678 \quad (26)$$

Using this value in eq. (8), the resonance energies of the doubly excited  ${}_{\text{N}}(\text{K}, \text{T})_n^A {}^{1,3}\text{P}^0$  states of the helium isoelectronic sequence are expressed explicitly as follows

For the Nsnp  ${}^1\text{P}^0$  doubly excited states, total spin  $S = 0$  and the electron spin  $s = 0,5$

$$E = -Z^2 \left\{ \frac{1}{N^2} + \frac{1}{n^2} \left( 1 - \frac{0,40678 \times ((n-K)-0,5)}{Z(n-1)} - \frac{0,40678}{Z} \right)^2 \right\} \text{Ryb} \quad (27)$$

For the Nsnp  ${}^3\text{P}^0$  doubly excited states, total spin  $S = 1$  and the electron spin  $s = 0,5$

$$E = -Z^2 \left\{ \frac{1}{N^2} + \frac{1}{n^2} \left( 1 - \frac{0,40678 \times ((n-K)-1,25)}{Z(n-1)} - \frac{0,40678}{Z} \right)^2 \right\} \text{Ryb} \quad (28)$$

At this present work, one can appreciate positively the merit of the variationnal procedure of the SCUNC formalism as energies of the doubly excited  ${}_{\text{N}}(\text{K}, \text{T})_n^A {}^{1,3}\text{P}^0$  excited states of the helium isoelectronic sequence can be easily calculated directly without using any computational code. The resonance energies and excitation energies of the doubly  ${}_{\text{N}}(\text{K}, 1)_n^A {}^{1,3}\text{P}^0$  excited states of the helium isoelectronic sequence obtained in the present calculations are listed in Tables 1-10.

In Table 11, we have indicated the correspondence between various classification schemes for autoionizing states in two electron systems [18-20, 10].

In Table 12, the present results for resonance energies of the doubly excited  ${}_2(0,1)_3^+ {}^1\text{P}^0$ ,  ${}_2(0,1)_4^+ {}^1\text{P}^0$  and  ${}_3(0,1)_4^+ {}^1\text{P}^0$  states of He-like systems ( $Z = 2 - 10$ ) are compared with those obtained from the semi-empirical procedure of the SCUNC formalism by Sakho *et al.* [13], Sakho [14], the variational method with the no-linear parameters of Hylleraas combined by the  $\beta$ -parameters of screening constant by unit nuclear charge by Dieng *et al.* [6], the complex-coordinate rotation method by Ho [3, 4], the truncated diagonalization method by Lipsky *et al.* [9], the stabilization method by Kar and Ho [22], the computing double sum over the complete hydrogen spectrum by Ivanov and Safronova [7] and the Fechbach projection formalism by Bachau *et al.* [8]. Comparison shows that the present SCUNC results are generally in good

agreement with those obtained by the aforementioned works.

Table 13, shows a comparison of the present SCUNC results of resonance energies of the doubly excited  ${}_2(0,1)_3^- {}^3\text{P}^0$ ,  ${}_2(0,1)_4^- {}^3\text{P}^0$  and  ${}_3(0,1)_4^- {}^3\text{P}^0$  states of the He-like systems ( $Z = 2 - 10$ ) with those obtained by Dieng *et al.* [6], Sakho [17], Ho [4], Lipsky *et al.* [9], Ivanov and Safronova [7], Sakho *et al.* [13] and Bachau *et al.* [8]. The agreements between the calculations are seen to be good.

Tables 14-19 lists the present SCUNC results for resonance energies of the doubly excited  ${}_{\text{N}}(\text{K}, 1)_n^A {}^{1,3}\text{P}^0$  ( $N = 2 - 10$ ;  $n = 5 - 10$ ) states of helium-like ions ( $Z = 2 - 10$ ) of He-like systems compared with those obtained by Dieng *et al.* [6]. A comparative reading of the results recorded in their tables shows a good agreement between the present SCUNC results and those of the listed literature data of Dieng *et al.* [6].

The results for the excitation energies of the  ${}_2(0,1)_n^+ {}^{1,3}\text{P}^0$ ,  ${}_3(1,1)_n^+ {}^{1,3}\text{P}^0$  levels of He I and Li II are listed in Tables 20 and 21 and compared with theoretical results of Dieng *et al.* [6], with the photoionization experimental results of Domke *et al.* [23], with the electron-impact experiment of Hicks and Comer [24], with photoabsorption experiments of Dehz and Ederer [25], Madden and Codling [26,27], Diehl *et al.* [28], with advanced light source experiments (ALS) of (Schulz *et al.* [29], Scully *et al.* [30], Rudd [31,32]). Here we note a

good agreement between the present theoretical data and the experimental observations. Our results are obtained using the experimental ground state energy  $-79.01\text{ eV}$  of He I and  $-198.09\text{ eV}$  of Li II [33, 34]. For energy conversion the infinite Rydberg is used ( $1\text{Ry} = 13.60569\text{ eV}$ ).

In Tables 22 and 23, the present results for resonance energies of the doubly excited  ${}^3(1,1)_n^{+}\text{P}^0$ ,  ${}^3(1,1)_n^{-}\text{P}^0$  ( $n = 4 - 10$ ) states of helium-like ions ( $Z = 2 - 40$ ) are compared with those obtained from the semi-empirical procedure of the SCUNC formalism by Sakho [16], with the Feshbach formalism (FF) computations of Bachau *et al.* [8]. The agreements are seen to be very good.

Table 24 shows the comparison of the present calculations for the energy positions of the  ${}^3(N, 1)_n^{+}\text{P}^0$  intershell states of He with the semi-empirical procedure of the SCUNC formalism by Sakho [15], the complex-coordinate rotation results of Ho [4, 5, 2], the hyperspherical coordinate values of (Sadeghpour [35], Koyama *et al.* [36]), the complex-coordinate scaling results of Burgers and Lindroth [37], the multiconfigurational Hartree-Fock calculations of Nicolaides and Komninos [38], and with the photodetachment experimental data of Cohen *et al.* [39]. In general, our results agree very well with the listed literature values.

Table 25 lists excitation energies ( $E$ , eV) for  ${}^3(1,1)_4^{+}\text{P}^0$  states of He-like ( $Z = 2 - 10$ ). The very good agreements between the present results predictions and both semi-empirical procedure of the SCUNC formalism of Sakho [11], diagonalization values of Wagué [40], complex scaling results of Ho [1] and hyperspherical coordinate data of Tang *et al.*, [41] give additional credit in the present calculations.

Overall, the good agreements between the present

calculations and the various *ab initio* results for the doubly  $N(K, T)_n^{A-1,3}\text{P}^0$  excited states of the helium isoelectronic sequence, justify the validity of the present variational procedure of the Screening Constant by Unit Nuclear Charge formalism. It should be mentioned that, calculations are directly obtained from analytical formulae in contrast with all the *ab initio* methods cited in this paper.

On the other hand, interpreting the ( $K, T, A$ ) correlation quantum numbers, Lin [21] stated that, for states with positive  $K$ , the two electrons tend to stay on the opposite sides of the nucleus while in states with negative  $K$  the two electrons tend to stay on the same side of the nucleus. This statement can be verified quantitatively in the framework of the SCUNC formalism by calculating the radial expectations values  ${}_N\langle r_{12}^{-1} \rangle_n$  given by (in a.u) [16]

$${}_N\langle r_{12}^{-1} \rangle_n = \frac{Z^2}{2N^2} + \frac{Z^2}{2n^2} - E({}_N(K, T)_n^{A-2S+1}L^\pi) \quad (29)$$

Then

- For states with positive  $K$ , the two electrons tend to stay on the opposite sides of the nucleus, as a result the interelectronic distance  $r_{12}$  increases and  ${}_N\langle r_{12}^{-1} \rangle_n$  decreases.

- For states with negative  $K$  the two electrons tend to stay on the same side of the nucleus. The interelectronic distance  $r_{12}$  decreases while  ${}_N\langle r_{12}^{-1} \rangle_n$  increases. So, in Table 26 the present results of radial expectation values  ${}_3\langle r_{12}^{-1} \rangle_n$  (in a. u) for some  ${}^3(N, 1)_n^{+}\text{P}^0$  Rydberg series of He-like systems ( $Z = 2 - 10$ ) are compared with those obtained by the semi-empirical procedure of the SCUNC formalism Screening constant by unit nuclear charge (SCUNC).

**Table 1.** Energy resonances of doubly excited  ${}^3(0,1)_n^{+}\text{P}^0$  and  ${}^3(0,1)_n^{-}\text{P}^0$  states of helium-like ions ( $Z = 2 - 10$ ). The results are expressed in Rydberg:  $1\text{Ry} = 13.60569\text{ eV}$ .

$\begin{matrix} Z \\ n \end{matrix}$	2	3	4	5	6	7	8	9	10
${}^3(0,1)_n^{+}\text{P}^0$									
3	1.1307	2.7329	5.0573	8.1039	11.8727	16.3637	21.5770	27.5125	34.1702
4	1.0782	2.5305	4.6079	7.3102	10.6375	14.5898	19.1672	24.3695	30.1968
5	1.0516	2.4324	4.3933	6.9341	10.0550	13.7558	18.0366	22.8975	28.3383
6	1.0365	2.3779	4.2749	6.7274	9.7355	13.2992	17.4184	22.0931	27.3234
7	1.0271	2.3446	4.2028	6.6019	9.5418	13.0225	17.0440	21.6064	26.7096
8	1.0209	2.3227	4.1558	6.5201	9.4156	12.8424	16.8004	21.2897	26.3103
9	1.0166	2.3077	4.1234	6.4637	9.3288	12.7186	16.6331	21.0722	26.0361
10	1.0135	2.2968	4.1001	6.4234	9.2666	12.6299	16.5132	20.9165	25.8398
11	1.0112	2.2888	4.0828	6.3934	9.2206	12.5642	16.4244	20.8011	25.6944
12	1.0095	2.2826	4.0697	6.3706	9.1855	12.5142	16.3568	20.7133	25.5837
13	1.0081	2.2778	4.0594	6.3529	9.1581	12.4752	16.3041	20.6449	25.4975
14	1.0070	2.2740	4.0513	6.3388	9.1364	12.4443	16.2623	20.5906	25.4291
15	1.0061	2.2710	4.0447	6.3274	9.1189	12.4193	16.2286	20.5468	25.3739
${}^3(0,1)_n^{-}\text{P}^0$									
3	1.1701	2.8061	5.1644	8.2449	12.0476	16.5726	21.8198	27.7891	34.4808
4	1.0931	2.5581	4.6481	7.3632	10.7032	14.6683	19.2583	24.4733	30.3134
5	1.0587	2.4457	4.4126	6.9596	10.0865	13.7935	18.0804	22.9474	28.3943
6	1.0405	2.3853	4.2856	6.7416	9.7531	13.3201	17.4427	22.1208	27.3546
7	1.0296	2.3491	4.2094	6.6106	9.5525	13.0353	17.0589	21.6234	26.7286
8	1.0225	2.3257	4.1601	6.5257	9.4227	12.8508	16.8102	21.3009	26.3228
9	1.0178	2.3097	4.1263	6.4677	9.3337	12.7244	16.6398	21.0799	26.0447
10	1.0143	2.2983	4.1023	6.4262	9.2702	12.6341	16.5181	20.9220	25.8460
11	1.0118	2.2899	4.0844	6.3955	9.2232	12.5673	16.4280	20.8052	25.6990
12	1.0099	2.2835	4.0709	6.3722	9.1875	12.5166	16.3596	20.7164	25.5872
13	1.0084	2.2785	4.0604	6.3541	9.1597	12.4771	16.3063	20.6474	25.5003
14	1.0073	2.2746	4.0521	6.3398	9.1377	12.4458	16.2641	20.5926	25.4313
15	1.0063	2.2714	4.0453	6.3282	9.1199	12.4205	16.2300	20.5484	25.3757

**Table 2.** Energy resonances of doubly excited  ${}^3(1,1)_n^{+} {}^1P^0$  and  ${}^3(1,1)_n^{-} {}^3P^0$  states of helium-like ions ( $Z = 2 - 10$ ). The results are expressed in Rydberg:  $1Ry = 13.60569 \text{ eV}$ .

$n \backslash Z$	2	3	4	5	6	7	8	9	10
${}^3(1,1)_n^{+} {}^1P^0$									
4	0.5428	1.3176	2.4396	3.9089	5.7254	7.8891	10.4001	13.2582	16.4636
5	0.5057	1.2002	2.1970	3.4959	5.0972	7.0006	9.2062	11.7141	14.5242
6	0.4863	1.1378	2.0671	3.2741	4.7590	6.5216	8.5620	10.8801	13.4761
7	0.4748	1.1006	1.9894	3.1413	4.5562	6.2341	8.1750	10.3790	12.8461
8	0.4675	1.0767	1.9393	3.0554	4.4250	6.0481	7.9246	10.0546	12.4381
9	0.4626	1.0604	1.9051	2.9968	4.3353	5.9208	7.7532	9.8325	12.1587
10	0.4591	1.0488	1.8808	2.9549	4.2713	5.8300	7.6308	9.6739	11.9592
11	0.4565	1.0402	1.8628	2.9240	4.2241	5.7628	7.5403	9.5566	11.8116
12	0.4545	1.0338	1.8491	2.9006	4.1881	5.7118	7.4716	9.4675	11.6995
13	0.4530	1.0287	1.8385	2.8823	4.1602	5.6721	7.4181	9.3982	11.6123
14	0.4518	1.0247	1.8301	2.8679	4.1381	5.6407	7.3758	9.3432	11.5431
15	0.4509	1.0215	1.8233	2.8562	4.1202	5.6153	7.3416	9.2989	11.4874
${}^3(1,1)_n^{-} {}^3P^0$									
4	0.5593	1.3469	2.4817	3.9636	5.7929	7.9693	10.4929	13.3638	16.5819
5	0.5135	1.2141	2.2170	3.5220	5.1293	7.0389	9.2506	11.7646	14.5808
6	0.4905	1.1454	2.0781	3.2886	4.7768	6.5428	8.5866	10.9081	13.5075
7	0.4774	1.1053	1.9962	3.1501	4.5670	6.2470	8.1901	10.3962	12.8653
8	0.4692	1.0797	1.9437	3.0612	4.4321	6.0566	7.9345	10.0658	12.4506
9	0.4637	1.0625	1.9082	3.0008	4.3403	5.9267	7.7600	9.8403	12.1674
10	0.4599	1.0503	1.8829	2.9578	4.2749	5.8342	7.6357	9.6795	11.9654
11	0.4571	1.0414	1.8644	2.9262	4.2267	5.7660	7.5440	9.5608	11.8163
12	0.4550	1.0346	1.8503	2.9022	4.1901	5.7142	7.4744	9.4707	11.7031
13	0.4534	1.0294	1.8395	2.8836	4.1618	5.6740	7.4203	9.4007	11.6151
14	0.4521	1.0253	1.8309	2.8689	4.1393	5.6422	7.3775	9.3452	11.5453
15	0.4511	1.0220	1.8239	2.8570	4.1212	5.6165	7.3430	9.3005	11.4891

**Table 3.** Energy resonances of doubly excited  ${}^4(2,1)_n^{+} {}^1P^0$  and  ${}^4(2,1)_n^{-} {}^3P^0$  states of helium-like ions ( $Z = 2 - 10$ ). The results are expressed in Rydberg:  $1Ry = 13.60569 \text{ eV}$ .

$n \backslash Z$	2	3	4	5	6	7	8	9	10
${}^4(2,1)_n^{+} {}^1P^0$									
5	0.3217	0.7813	1.4459	2.3156	3.3902	4.6698	6.1544	7.8440	9.7386
6	0.2976	0.7105	1.3040	2.0781	3.0328	4.1680	5.4837	6.9800	8.6569
7	0.2839	0.6693	1.2206	1.9378	2.8207	3.8694	5.0840	6.4644	8.0106
8	0.2753	0.6433	1.1674	1.8479	2.6845	3.6775	4.8266	6.1321	7.5937
9	0.2697	0.6257	1.1314	1.7868	2.5919	3.5467	4.6512	5.9054	7.3092
10	0.2657	0.6133	1.1059	1.7435	2.5261	3.4537	4.5262	5.7438	7.1064
11	0.2629	0.6042	1.0872	1.7116	2.4776	3.3851	4.4341	5.6246	6.9567
12	0.2607	0.5974	1.0730	1.6875	2.4408	3.3331	4.3642	5.5342	6.8431
13	0.2591	0.5921	1.0620	1.6687	2.4123	3.2927	4.3099	5.4640	6.7549
14	0.2578	0.5879	1.0533	1.6539	2.3897	3.2607	4.2669	5.4084	6.6850
15	0.2567	0.5846	1.0464	1.6420	2.3716	3.2350	4.2323	5.3635	6.6286
${}^4(2,1)_n^{-} {}^3P^0$									
5	0.3301	0.7958	1.4666	2.3423	3.4230	4.7087	6.1994	7.8951	9.7959
6	0.3021	0.7185	1.3154	2.0928	3.0509	4.1894	5.5086	7.0083	8.6885
7	0.2866	0.6741	1.2275	1.9467	2.8317	3.8825	5.0992	6.4816	8.0299
8	0.2771	0.6464	1.1719	1.8537	2.6918	3.6860	4.8366	6.1434	7.6064
9	0.2709	0.6279	1.1345	1.7909	2.5969	3.5526	4.6580	5.9132	7.3180
10	0.2666	0.6149	1.1081	1.7464	2.5296	3.4579	4.5312	5.7494	7.1127
11	0.2635	0.6054	1.0888	1.7137	2.4802	3.3882	4.4378	5.6288	6.9614
12	0.2612	0.5983	1.0742	1.6891	2.4428	3.3355	4.3670	5.5374	6.8467
13	0.2594	0.5928	1.0630	1.6700	2.4139	3.2946	4.3121	5.4665	6.7577
14	0.2581	0.5885	1.0541	1.6550	2.3910	3.2622	4.2687	5.4103	6.6872
15	0.2570	0.5850	1.0470	1.6428	2.3726	3.2362	4.2337	5.3651	6.6304

**Table 4.** Energy resonances of doubly excited  $\zeta(3,1)_n^+ \text{P}^0$ ,  $\zeta(3,1)_n^- \text{P}^0$ , and  $\zeta(4,1)_n^+ \text{P}^0$  states of helium-like ions ( $Z = 2 - 10$ ). The results are expressed in Rydberg:  $1\text{Ry} = 13.60569 \text{ eV}$ .

$n \backslash Z$	2	3	4	5	6	7	8	9	10
$\zeta(3,1)_n^+ \text{P}^0$									
6	0.2137	0.5186	0.9592	1.5353	2.2469	3.0942	4.0769	5.1952	6.4491
7	0.1975	0.4733	0.8698	1.3872	2.0254	2.7844	3.6643	46649	5.7864
8	0.1877	0.4449	0.8134	1.2932	1.8842	2.5864	3.3999	4.3246	5.3606
9	0.1813	0.4261	0.7755	1.2297	1.7886	24521	3.2203	4.0933	5.0709
10	0.1769	0.4129	0.7489	1.1848	1.7208	2.3568	3.0928	3.9288	4.8648
11	0.1737	0.4033	0.7294	1.1520	1.6711	2.2868	2.9990	3.8077	4.7130
12	0.1714	0.3961	0.7147	1.1271	1.6335	2.2338	2.9279	3.7160	4.5979
13	0.1696	0.3905	0.7033	1.1079	1.6044	2.1927	2.8728	3.6448	4.5086
14	0.1682	0.3862	0.6944	1.0928	1.5814	2.1602	2.8293	3.5885	4.4379
15	0.1670	0.3827	0.6872	1.0806	1.5629	2.1341	2.7942	3.5432	4.3810
$\zeta(3,1)_n^- \text{P}^0$									
6	0.2185	0.5268	0.9708	1.5503	2.2653	3.1159	4.1021	5.2238	6.4811
7	0.2004	0.4782	0.8769	1.3963	2.0366	2.7977	3.6796	4.6823	5.8059
8	0.1896	0.4482	0.8180	1.2991	1.8915	2.5951	3.4099	4.3360	5.3734
9	0.1826	0.4283	0.7787	1.2338	1.7936	2.4581	3.2272	41011	5.0797
10	0.1778	0.4144	0.7511	11878	1.7244	2.3611	3.0978	3.9344	4.8711
11	0.1744	0.4044	0.7310	1.1541	1.6738	2.2900	3.0027	3.8119	4.7177
12	0.1719	0.3969	0.7159	1.1288	1.6356	2.2362	2.9307	3.7192	4.6015
13	0.1699	0.3912	0.7043	1.1092	1.6060	2.1946	2.8750	3.6473	4.5114
14	0.1685	0.3867	0.6952	1.0938	1.5827	2.1617	2.8310	3.5905	4.4401
15	0.1673	0.3831	0.6878	1.0814	1.5639	2.1353	2.7956	3.5448	4.3828
$\zeta(4,1)_n^+ \text{P}^0$									
7	0.1525	0.3699	0.6837	1.0938	1.6003	2.2032	2.9025	3.6981	4.5901
8	0.1413	0.3392	0.6240	0.9955	1.4539	1.9991	2.6310	3.3498	4.1554
9	0.1341	0.3190	0.5842	0.9296	1.3553	1.8612	2.4473	3.1137	3.8604
10	0.1292	0.3050	0.5563	0.8832	1.2856	1.7637	2.3172	2.9463	3.6510
11	0.1257	0.2948	0.5360	0.8493	1.2347	1.6921	2.2217	2.8233	3.4970
12	0.1231	0.2872	0.5208	0.8238	1.1962	1.6381	2.1495	2.7302	3.3805
13	0.1212	0.2814	0.5091	0.8041	1.1665	1.5963	2.0935	2.6581	3.2901
14	0.1197	0.2769	0.4999	0.7886	1.1431	1.5633	2.0494	2.6011	3.2187
15	0.1185	0.2733	0.4925	0.7762	1.1243	1.5368	2.0139	2.5553	3.1612

**Table 5.** Energy resonances of doubly excited  $\zeta(4,1)_n^- \text{P}^0$ ,  $\zeta(5,1)_n^+ \text{P}^0$ , and  $\zeta(5,1)_n^- \text{P}^0$  states of helium-like ions ( $Z = 2 - 10$ ). The results are expressed in Rydberg:  $1\text{Ry} = 13.60569 \text{ eV}$ .

$n \backslash Z$	2	3	4	5	6	7	8	9	10
$\zeta(4,1)_n^- \text{P}^0$									
7	0.1555	0.3750	0.6908	1.1030	1.6116	2.2166	2.9180	3.7157	4.6098
8	0.1432	0.3425	0.6286	10016	1.4613	2.0078	2.6411	33613	4.1682
9	0.1354	0.3213	0.5874	0.9337	1.3603	1.8672	2.4543	3.1216	3.8692
10	0.1301	0.3066	0.5586	0.8862	1.2893	1.7680	2.3222	2.9520	3.6573
11	0.1264	0.2960	0.5377	0.8515	1.2374	1.6953	2.2254	2.8275	3.5017
12	0.1236	0.2881	0.5221	08255	1.1983	1.6406	2.1523	2.7334	3.3840
13	0.1216	0.2821	0.5101	0.8054	1.1681	1, 5982	2.0957	26606	3.2929
14	0.1200	0.2774	0.5007	0.7896	1.1444	15649	2.0511	2.6031	3.2209
15	0.1187	0.2737	0.4931	0.7770	1.1253	1.5381	2.0153	2.5569	3.1630
$\zeta(5,1)_n^+ \text{P}^0$									
8	0.1144	0.2773	0.5123	0.8193	1.1984	1.6496	2.1729	2.7682	3.4356
9	0.1064	0.2557	0.4705	0.7509	1.0967	1.5081	1.9850	2.5273	3.1352
10	0.1009	0.2408	0.4414	07029	1.0252	1.4083	1.8522	2.3569	2.9225
11	0.0971	0.2301	0.4204	0.6680	0.9730	1.3353	1.7550	2.2320	2.7663
12	0.0943	0.2221	0.4046	0.6418	0.9337	1.2803	1.6816	2.1376	2.6483
13	0.0922	0.2160	0.3925	0.6216	0.9034	1.2378	1.6248	2.0645	2.5569
14	0.0906	0.2113	0.3830	0.6057	0.8795	1.2043	1.5800	2.0068	2.4847
15	0.0893	0.2075	0.3754	0.5930	0.8603	1.1774	1.5441	1.9605	2.4266
$\zeta(5,1)_n^- \text{P}^0$									
8	0.1164	0.2807	0.5170	0.8254	1.2059	1.6584	2.1830	2.7797	3.4485
9	0.1077	0.2580	0.4738	0.7550	1.1018	1.5141	1.9920	2.5353	3.1441
10	0.1019	0.2424	0.4437	0.7059	1.0289	1.4126	1.8572	2.3626	2.9289
11	0.0978	0.2313	0.4221	0.6702	0.9757	1.3385	1.7587	2.2362	2.7711
12	0.0949	0.2230	0.4059	0.6435	0.9357	1.2827	1.6844	2.1408	2.6519
13	0.0926	0.2168	0.3935	0.6229	0.9050	1.2397	1.6270	2.0670	2.5597
14	0.0909	0.2119	0.3838	0.6068	0.8808	1.2058	1.5818	2.0088	2.4869
15	0.0896	0.2080	0.3761	0.5939	0.8614	1.1786	1.5455	1.9621	2.4284

**Table 6.** Energy resonances of doubly excited  ${}_8(6,1)_n^+ \text{P}^0$ ,  ${}_8(6,1)_n^- \text{P}^0$ ,  ${}_9(7,1)_n^+ \text{P}^0$  and  ${}_9(7,1)_n^- \text{P}^0$  states of helium-like ions ( $Z = 2 - 10$ ). The results are expressed in Rydberg:  $1\text{Ry} = 13.60569 \text{ eV}$ .

$n \backslash Z$	2	3	4	5	6	7	8	9	10
${}_8(6,1)_n^+ \text{P}^0$									
9	0.0890	0.2157	0.3983	0.6369	0.9314	1.2818	1.6882	2.1505	2.6688
10	0.0831	0.1999	0.3680	0.5873	0.8579	1.1797	1.5528	1.9771	2.4527
11	0.0789	0.1886	0.3461	0.5514	0.8044	1.1052	1.4538	1.8502	2.2943
12	0.0759	0.1803	0.3298	0.5244	0.7642	1.0492	1.3792	1.7544	2.1748
13	0.0737	0.1739	0.3173	0.5038	0.7333	1.0059	1.3217	1.6804	2.0823
14	0.0719	0.1690	0.3075	0.4875	0.7090	0.9719	1.2763	1.6221	2.0093
15	0.0706	0.1651	0.2998	0.4746	0.6895	0.9446	1.2399	1.5752	1.9507
${}_8(6,1)_n^- \text{P}^0$									
9	0.0904	0.2180	0.4016	0.6411	0.9365	1.2879	1.6952	2.1585	2.6777
10	0.0841	0.2016	0.3703	0.5903	0.8616	1.1841	1.5578	1.9828	2.4591
11	0.0797	0.1899	0.3478	0.5536	0.8071	1.1085	1.4575	1.8544	2.2991
12	0.0765	0.1812	0.3311	0.5261	0.7663	1.0516	1.3821	1.7577	2.1784
13	0.0741	0.1747	0.3183	0.5051	0.7349	1.0079	1.3239	1.6830	2.0851
14	0.0722	0.1696	0.3084	0.4886	0.7103	0.9734	1.2780	1.6241	2.0116
15	0.0708	0.1655	0.3004	0.4754	0.6906	0.9459	1.2413	1.5768	1.9525
${}_9(7,1)_n^+ \text{P}^0$									
10	0.0713	0.1726	0.3186	0.5094	0.7448	1.0249	1.3497	1.7191	2.1333
11	0.0668	0.1608	0.2959	0.4724	0.6900	0.9488	1.2489	1.5902	1.9727
12	0.0635	0.1520	0.2791	0.4447	0.6490	0.8918	1.1732	1.4931	1.8517
13	0.0611	0.1454	0.2662	0.4235	0.6174	0.8478	1.1147	1.4182	1.7582
14	0.0592	0.1402	0.2562	0.4070	0.5927	0.8133	1.0687	1.3591	1.6844
15	0.0578	0.1362	0.2482	0.3937	0.5729	0.7856	1.0319	1.3118	1.6252
${}_9(7,1)_n^- \text{P}^0$									
10	0.0723	0.1743	0.3210	0.5124	0.7485	1.0293	1.3547	1.7249	2.1397
11	0.0675	0.1620	0.2977	0.4746	0.6928	0.9521	1.2527	1.5945	1.9775
12	0.0641	0.1529	0.2804	0.4464	0.6511	0.8942	1.1760	1.4964	1.8553
13	0.0615	0.1461	0.2672	0.4249	0.6191	0.8497	1.1170	1.4207	1.7610
14	0.0596	0.1408	0.2570	0.4080	0.5940	0.8148	1.0705	1.3611	1.6867
15	0.0581	0.1366	0.2488	0.3946	0.5739	0.7868	1.0333	1.3134	1.6270

**Table 7.** Energy resonances of doubly excited  ${}_{10}(8,1)_n^+ \text{P}^0$ ,  ${}_{10}(8,1)_n^- \text{P}^0$ ,  ${}_{11}(9,1)_n^+ \text{P}^0$  and  ${}_{11}(9,1)_n^- \text{P}^0$  states of helium-like ions ( $Z = 2 - 10$ ). The results are expressed in Rydberg:  $1\text{Ry} = 13.60569 \text{ eV}$ .

$n \backslash Z$	2	3	4	5	6	7	8	9	10
${}_{10}(8,1)_n^+ \text{P}^0$									
11	0.0584	0.1413	0.2607	0.4167	0.6092	0.8383	1.1038	1.4059	1.7445
12	0.0549	0.1322	0.2433	0.3884	0.5673	0.7801	1.0269	1.3075	1.6220
13	0.0523	0.1252	0.2300	0.3667	0.5352	0.7355	0.9676	1.2316	1.5274
14	0.0503	0.1199	0.2197	0.3497	0.5099	0.7004	0.9210	1.1718	1.4528
15	0.0464	0.1115	0.2055	0.3284	0.4802	0.6609	0.8705	1.1089	1.3763
${}_{10}(8,1)_n^- \text{P}^0$									
11	0.0591	0.1426	0.2625	0.4190	0.6120	0.8415	1.1076	1.4102	1.7493
12	0.0554	0.1331	0.2447	0.3901	0.5694	0.7826	1.0297	1.3107	1.6256
13	0.0527	0.1260	0.2311	0.3680	0.5368	0.7374	0.9698	1.2341	1.5302
14	0.0506	0.1205	0.2205	0.3508	0.5112	0.7019	0.9228	1.1738	1.4551
15	0.0490	0.1162	0.2122	0.3371	0.4909	0.6735	0.8851	1.1256	1.3949
${}_{11}(9,1)_n^+ \text{P}^0$									
12	0.0487	0.1178	0.2173	0.3473	0.5076	0.6984	0.9196	1.1712	1.4533
13	0.0459	0.1106	0.2037	0.3251	0.4749	0.6530	0.8595	1.0944	1.3576
14	0.0438	0.1051	0.1930	0.3078	0.4492	0.6174	0.8123	1.0339	1.2823
15	0.0422	0.1007	0.1846	0.2940	0.4287	0.5889	0.7745	0.9855	1.2220
${}_{11}(9,1)_n^- \text{P}^0$									
12	0.0493	0.1188	0.2187	0.3490	0.5098	0.7009	0.9225	1.1745	1.4569
13	0.0464	0.1114	0.2047	0.3264	0.4765	0.6550	0.8618	1.0969	1.3605
14	0.0442	0.1057	0.1939	0.3088	0.4505	0.6189	0.8141	1.0360	1.2846
15	0.0425	0.1012	0.1853	0.2948	0.4298	0.5902	0.7759	0.9872	1.2238

**Table 8.** Energy resonances of doubly excited  ${}_{12}(10,1)_n^+ {}^1P^0$ ,  ${}_{12}(10,1)_n^- {}^3P^0$ ,  ${}_{13}(11,1)_n^+ {}^1P^0$ ,  ${}_{13}(11,1)_n^- {}^3P^0$ ,  ${}_{14}(12,1)_{15}^+ {}^1P^0$  and  ${}_{14}(12,1)_{15}^- {}^3P^0$  states of helium-like ions ( $Z = 2 - 10$ ). The results are expressed in Rydberg:  $1Ry = 13.60569$  eV.

$n \backslash Z$	2	3	4	5	6	7	8	9	10
${}_{12}(10,1)_n^+ {}^1P^0$									
13	0.0412	0.0997	0.1839	0.2939	0.4295	0.5909	0.7780	0.9909	1.2294
14	0.0390	0.0940	0.1730	0.2762	0.4034	0.5548	0.7302	0.9297	1.1533
15	0.0373	0.0894	0.1644	0.2621	0.3826	0.5259	0.6919	0.8808	1.0924
13	0.0417	0.1005	0.1850	0.2952	0.4312	0.5929	0.7803	0.9934	1.2323
14	0.0394	0.0946	0.1739	0.2773	0.4047	0.5563	0.7320	0.9317	1.1556
15	0.0376	0.0899	0.1651	0.2630	0.3837	0.5271	0.6934	0.8824	1.0942
${}_{13}(11,1)_n^+ {}^1P^0$									
14	0.0354	0.0855	0.1577	0.2519	0.3682	0.5065	0.6668	0.8492	1.0536
15	0.0336	0.0808	0.1489	0.2376	0.3470	0.4772	0.6281	0.7998	0.9921
${}_{13}(11,1)_n^- {}^3P^0$									
14	0.0357	0.0861	0.1586	0.2530	0.3695	0.5081	0.6686	0.8513	1.0559
15	0.0338	0.0813	0.1495	0.2385	0.3481	0.4785	0.6296	0.8014	0.9940
${}_{14}(12,1)_{15}^+ {}^1P^0$									
15	0.0307	0.0742	0.1367	0.2184	0.3191	0.4390	0.5779	0.7359	0.9131
${}_{14}(12,1)_{15}^- {}^3P^0$									
15	0.0310	0.0746	0.1374	0.2193	0.3202	0.4402	0.5794	0.7376	0.9149

**Table 9.** Excitation energy of  ${}_2(0,1)_n^+ {}^1P^0$  and  ${}_2(0,1)_n^- {}^3P^0$  states of helium-like ions ( $Z = 2 - 10$ ) below the  $n = 3 - 15$ . The results are expressed in eV:  $1Ry = 13.60569$  eV.

$n \backslash Z$	2	3	4	5	6	7	8	9	10
${}_2(0,1)_n^+ {}^1P^0$									
3	63.63	160.91	302.78	489.23	720.28	995.92	1315.83	1680.99	2090.42
4	64.34	163.66	308.90	500.03	737.09	1020.06	1348.62	1723.76	2144.48
5	64.70	165.00	311.82	505.15	745.02	1031.40	1364.00	1743.78	2169.77
6	64.91	165.74	313.43	507.96	749.36	1037.62	1372.41	1754.73	2183.58
7	65.04	166.19	314.41	509.67	752.00	1041.38	1377.50	1761.35	2191.93
8	65.12	166.49	315.05	510.78	753.71	1043.83	1380.82	1765.66	2197.36
9	65.18	166.69	315.49	511.55	754.89	1045.51	1383.10	1768.62	2201.09
10	65.22	166.84	315.81	512.10	755.74	1046.72	1384.73	1770.74	2203.76
11	65.25	166.95	316.04	512.50	756.37	1047.62	1385.93	1772.31	2205.74
12	65.28	167.03	316.22	512.81	756.85	1048.30	1386.85	1773.50	2207.25
13	65.29	167.10	316.36	513.05	757.22	1048.83	1387.57	1774.43	2208.42
14	65.31	167.15	316.47	513.25	757.51	1049.25	1388.14	1775.17	2209.35
15	65.32	167.19	316.56	513.40	757.75	1049.59	1388.60	1775.77	2210.10
${}_2(0,1)_n^- {}^3P^0$									
3	63.09	159.91	300.82	486.66	717.09	992.12	1311.41	1675.96	2084.77
4	64.14	163.29	308.16	499.06	735.89	1018.63	1346.96	1721.87	2142.36
5	64.61	164.81	311.46	504.68	744.44	1030.72	1363.20	1742.88	2168.75
6	64.85	165.64	313.23	507.70	749.04	1037.24	1371.97	1754.22	2183.01
7	65.00	166.13	314.29	509.51	751.80	1041.15	1377.23	1761.04	2191.58
8	65.10	166.45	314.97	510.68	753.59	1043.68	1380.64	1765.46	2197.13
9	65.16	166.66	315.43	511.47	754.81	1045.41	1382.97	1768.48	2200.93
10	65.21	166.82	315.77	512.04	755.68	1046.65	1384.64	1770.64	2203.65
11	65.24	166.93	316.01	512.46	756.32	1047.56	1385.87	1772.23	2205.66
12	65.27	167.02	316.20	512.78	756.81	1048.25	1386.80	1773.44	2207.18
13	65.29	167.09	316.34	513.03	757.19	1048.79	1387.53	1774.39	2208.37
14	65.31	167.14	316.46	513.23	757.49	1049.22	1388.11	1775.13	2209.31
15	65.32	167.19	316.55	513.39	757.73	1049.57	1388.57	1775.74	2210.07

**Table 10.** Excitation energy of  ${}_3(1,1)_n^+ {}^1P^0$  and  ${}_3(1,1)_n^- {}^3P^0$  states of helium-like ions ( $Z = 2 - 10$ ) below the  $n = 4 - 15$ . The results are expressed in eV:  $1Ry = 13.60569$  eV.

$n \backslash Z$	2	3	4	5	6	7	8	9	10
${}_3(1,1)_n^+ {}^1P^0$									
4	71.63	180.16	338.40	546.31	803.92	1111.22	1467.90	1874.93	2331.33
5	72.13	181.76	341.70	551.93	812.47	1123.31	1484.14	1895.94	2357.72
6	72.39	182.61	343.47	554.94	817.07	1129.83	1492.91	1907.29	2371.98
7	72.55	183.12	344.52	556.75	819.83	1133.74	1498.17	1914.11	2380.55
8	72.65	183.44	345.20	557.92	821.61	1136.27	1501.58	1918.52	2386.10
9	72.72	183.66	345.67	558.72	822.83	1138.00	1503.91	1921.54	2389.90
10	72.76	183.82	346.00	559.29	823.71	1139.24	1505.58	1923.70	2392.62
11	72.80	183.94	346.25	559.71	824.35	1140.15	1506.81	1925.30	2394.62

$n \setminus Z$	2	3	4	5	6	7	8	9	10
12	72.83	184.02	346.43	560.03	824.84	1140.85	1507.74	1926.51	2396.15
13	72.85	184.09	346.58	560.27	825.22	1141.39	1508.47	1927.45	2397.34
14	72.86	184.15	346.69	560.47	825.52	1141.81	1509.05	1928.20	2398.28
15	72.88	184.19	346.78	560.63	825.76	1142.16	1509.51	1928.80	2399.04
${}^3(1,1)_n^- {}^3\text{P}^0$									
4	71.40	179.76	337.83	545.56	803.00	1110.13	1466.64	1873.50	2329.72
5	72.02	181.57	34143	551.57	1812.03	1122.79	1483.54	1895.25	2356.95
6	72.34	182.51	343.32	554.75	1816.83	1129.54	1492.57	1906.91	2371.55
7	72.51	183.05	344.43	556.63	1819.68	1133.56	1497.97	1913.87	2380.29
8	72.63	183.40	345.14	557.84	1821.52	1136.16	1501.45	1918.37	2385.93
9	72.70	183.63	34563	558.66	1822.77	1137.92	1503.82	1921.44	2389.78
10	72.75	183.80	345.97	559.25	1823.66	1139.18	1505.51	1923.62	2392.53
11	72.79	183.92	346.22	559.68	1824.31	1140.11	1506.76	1925.24	2394.56
12	72.82	184.01	346.41	560.00	1824.81	1140.81	1507.71	1926.47	2396.10
13	72.84	18408	346.56	560.26	1825.20	1141.36	1508.44	1927.42	2397.30
14	72.86	184.14	346.68	560.46	1825.50	1141.79	1509.02	1928.17	2398.25
15	72.87	184.19	346.77	560.62	1825.75	1142.14	1509.49	1928.78	2399.01

**Table 11.** Correspondence between various classification schemes for autoionizing states in two-electron systems below the  $N = 2-9$  hydrogenic threshold. The  $S$  denotes the total spin angular momentum,  $L$  is the total orbital angular momentum,  $\pi$  represents the parity,  $N$  and  $n$  are respectively the principal quantum numbers of the inner and the outer electron.  $S$ ,  $L$ ,  $\pi$ ,  $N$  and  $n$  are common to all the schemes. The correlation quantum numbers  $K$ ,  $T$  and  $A$  used to label each state are introduced to complete the description.

Usual Classification $Nnl^l {}^{2S+1}L^\pi$	Conneely and Lipsky ( $N, n, \alpha$ )	Herrick and Sinanoglu $n(K, T)_n$	Lin A	Lowest $n$ of the series
2snp ${}^1\text{P}^0$	(2,n,a)	${}_2(0,1)_n$	+	2
2snp ${}^3\text{P}^0$	(2,n,a)	${}_2(0,1)_n$	-	2
3snp ${}^1\text{P}^0$	(3,n,a)	${}_3(1,1)_n$	+	3
3snp ${}^3\text{P}^0$	(3,n,a)	${}_3(1,1)_n$	-	3
4snp ${}^1\text{P}^0$	(4,n,a)	${}_4(2,1)_n$	+	4
4snp ${}^3\text{P}^0$	(4,n,a)	${}_4(2,1)_n$	-	4
5snp ${}^1\text{P}^0$	(5,n,a)	${}_5(3,1)_n$	+	5
5snp ${}^3\text{P}^0$	(5,n,a)	${}_5(3,1)_n$	-	5
6snp ${}^1\text{P}^0$	(6,n,a)	${}_6(4,1)_n$	+	6
6snp ${}^3\text{P}^0$	(6,n,a)	${}_6(4,1)_n$	-	6
7snp ${}^1\text{P}^0$	(7,n,a)	${}_7(5,1)_n$	+	7
7snp ${}^3\text{P}^0$	(7,n,a)	${}_7(5,1)_n$	-	7
8snp ${}^1\text{P}^0$	(8,n,a)	${}_8(6,1)_n$	+	8
8snp ${}^3\text{P}^0$	(8,n,a)	${}_8(6,1)_n$	-	8
9snp ${}^1\text{P}^0$	(9,n,a)	${}_9(7,1)_n$	+	9
9snp ${}^3\text{P}^0$	(9,n,a)	${}_9(7,1)_n$	-	9
10snp ${}^1\text{P}^0$	(10,n,a)	${}_{10}(8,1)_n$	+	10
10snp ${}^3\text{P}^0$	(10,n,a)	${}_{10}(8,1)_n$	-	10
11snp ${}^1\text{P}^0$	(11,n,a)	${}_{11}(9,1)_n$	+	11
11snp ${}^3\text{P}^0$	(11,n,a)	${}_{11}(9,1)_n$	-	11
12snp ${}^1\text{P}^0$	(12,n,a)	${}_{12}(10,1)_n$	+	12
12snp ${}^3\text{P}^0$	(12,n,a)	${}_{12}(10,1)_n$	-	12
13snp ${}^1\text{P}^0$	(13,n,a)	${}_{13}(11,1)_n$	+	13
13snp ${}^3\text{P}^0$	(13,n,a)	${}_{13}(11,1)_n$	-	13
14snp ${}^1\text{P}^0$	(14,n,a)	${}_{14}(12,1)_n$	+	14
14snp ${}^3\text{P}^0$	(14,n,a)	${}_{14}(12,1)_n$	-	14

**Table 12.** Comparison of doubly excited  ${}_2(0,1)_3^+ {}^1\text{P}^0$ ,  ${}_2(0,1)_4^+ {}^1\text{P}^0$  and  ${}_3(0,1)_4^+ {}^1\text{P}^0$  states of helium-like ions ( $Z = 2-10$ ) with other results. Energies  $E$  are in Ry.

$Z$	${}_2(0,1)_3^+ {}^1\text{P}^0$					
	Present results - $E$	Dieng <i>et al.</i> (2014) - $E$	Sakho (2011) - $E$	Ho (1996) - $E$	Lipsky <i>et al.</i> (1977) - $E$	Kar and Ho (2009) - $E$
2	1.1307	1.1245	1.1284	1.1280	1.1258	
3	2.7329	2.7209	2.7288	2.7228	2.7131	
4	5.0573	5.0394	5.0515	5.0573	5.0573	
5	8.1039	8.0802	8.0964	8.0707	8.0444	
6	11.8727	11.8432	11.8636	11.8263		11.8263

$^2(0,1)_3^+ \text{P}^0$						
Z	Present results -E	Dieng <i>et al.</i> (2014) -E	Sakho (2011) -E	Ho (1996) -E	Lipsky <i>et al.</i> (1977) -E	Kar and Ho (2009) -E
7	16.3637	16.3285	16.3530	16.3035		
8	21.5770	21.5359	21.5647	21.5026		21.5028
9	27.5125	27.4656	27.4986	27.4238		
10	34.1702	34.1175	34.1566	34.0669		34.0671

$^2(0,1)_4^+ \text{P}^0$						
Z	Present results -E	Dieng <i>et al.</i> (2014) -E	Sakho (2011) -E	Ivanov and Safronova (1993) -E	Lipsky <i>et al.</i> (1977) -E	Kar and Ho (2009) -E
2	1.0782	1.0892	1.0689	1.0815	1.0677	
3	2.5305	2.5510	2.5128	2.5341	2.5056	
4	4.6079	4.6378	4.5817	4.6117	4.5677	
5	7.3102	7.3496	7.2756	7.3142	7.2507	
6	10.6375	10.6864	10.5945	10.6418		10.5847
7	14.5898	14.6482	14.5384	14.5943		
8	19.1672	19.2350	19.1073	19.1719		19.0914
9	24.3695	24.4468	24.3013	24.3744		
10	30.1968	30.2836	30.1202	30.2020		30.0979

$^3(1,1)_3^+ \text{P}^0$						
Z	Present results -E	Dieng <i>et al.</i> (2014) -E	Sakho <i>et al.</i> (2010) -E	Bachau <i>et al.</i> (1991) -E	Lipsky <i>et al.</i> (1977) -E	Ho (1991) -E
2	0.5428	0.5494	0.5321	0.5424	0.5415	0.5424
3	1.3176	1.3294	1.3127	1.3196	1.3169	
4	2.4396	2.4566	2.4405	2.4440	2.4387	
5	3.9089	3.9311	3.9156	3.9140	3.9078	
6	5.7254	5.7528	5.7379	5.7300		
7	7.8891	7.9217	7.9074	7.8960		
8	10.4001	10.4378	10.4242	10.4080		
9	13.2582	13.3012	13.2881	13.2660		
10	16.4636	16.5117	16.4993	16.4720		

**Table 13.** Comparison of doubly excited  $^2(0,1)_3^- \text{P}^0$ ,  $^2(0,1)_4^- \text{P}^0$  and  $^3(0,1)_4^- \text{P}^0$  states of helium-like ions ( $Z = 2 - 10$ ) with other results. Energies E are in Ryb.

$^2(0,1)_3^- \text{P}^0$						
Z	Present results -E	Dieng <i>et al.</i> (2014) -E	Sakho (2011) -E	Ho (1996) -E	Lipsky <i>et al.</i> (1977) - E	
2	1.1701	1.1727	1.1722	1.1581		
3	2.8061	2.8109	2.8134	2.7970		
4	5.1644	5.1714	5.1774	5.1585		
5	8.2449	8.2540	8.2639	8.2422	1.1667	
6	12.0476	12.0588	12.0727	12.0482	2.8060	
7	16.5726	16.5859	16.6037	16.5763	5.1687	
8	21.8198	21.8352	21.8571	21.8267	8.2547	
9	27.7891	27.8068	27.8326	27.7993		
10	34.4808	34.5005	34.5305	34.4941		

$^2(0,1)_4^- \text{P}^0$						
Z	Present results -E	Dieng <i>et al.</i> (2014) -E	Sakho (2011) -E	Ivanov and Safronova (1993) -E	Lipsky <i>et al.</i> (1977) -E	
2	1.0981	1.1127	1.0860	1.0739		
3	2.5581	2.5931	2.5463	2.5304		
4	4.6481	4.6985	4.6317	4.6119	1.0844	
5	7.3632	7.4288	7.3422	7.3184	2.5452	
6	10.7032	10.7842	10.6777	10.6498	4.6318	
7	14.6683	14.7646	14.6383	14.6063	7.2965	
8	19.2583	19.3699	19.2238	19.1878		
9	24.4733	24.6003	24.4344	24.3942		
10	30.3134	30.4557	30.2700	30.2257		

$^3(1,1)_4^- \text{P}^0$						
Z	Present results -E	Dieng <i>et al.</i> (2014) -E	Sakho <i>et al.</i> (2010) -E	Bachau <i>et al.</i> (1991) -E	Lipsky <i>et al.</i> (1977) - E	
2	0.5593	0.5669	0.5388	0.5578	0.5591	
3	1.3469	1.3600	1.3197	1.3501	1.3506	
4	2.4817	2.5002	2.4476	2.4900	2.4889	
5	3.9636	3.9877	3.9228	3.9780	3.9749	

${}^3(1,1)_4^- {}^3P^0$					
Z	Present results -E	Dieng <i>et al.</i> (2014) -E	Sakho <i>et al.</i> (2010) -E	Bachau <i>et al.</i> (1991) -E	Lipsky <i>et al.</i> (1977) - E
6	5.7929	5.8224	5.74511	5.8120	
7	7.9693	8.0043	7.9147	7.9940	
8	10.4929	10.5335	10.4314	10.5220	
9	13.3638	13.4098	13.2954	13.3980	
10	16.5819	16.6334	16.5066	16.6220	

**Table 14.** Comparison of energy resonances ( $-E$ , Ryb) of doubly excited  ${}_2(0,1)_n^A {}^{1,3}P^o$  ( $n = 5 - 10$ ) states of helium-like ions ( $Z = 2 - 10$ ) with Dieng *et al.* (2014) theoretical results.

${}_2(0,1)_n^+ {}^1P^0$						
n	5	6	7			
Z	Present results	Dieng <i>et al.</i> (2014)	Present results	Dieng <i>et al.</i> (2014)	Present results	Dieng <i>et al.</i> (2014)
2	1.0516	1.0431	1,0365	1.0296	1,0271	1.0210
3	2.4324	2.4160	2,3779	2.3646	2,3446	2.3327
4	4.3933	4.3690	4,2749	4.2553	4,2028	4.1853
5	6.9341	6.9020	6,7274	6.7015	6,6019	6.5787
6	10.0550	10.0150	9,7355	9.7032	9,5418	9.5129
7	13.7558	13.7080	13,2992	13.2606	13,0225	12.9880
8	18.0366	17.9810	17,4184	17.3734	17,0440	17.0040
9	22.8975	22.8340	22,0931	22.0418	21,6064	21.5605
10	28.3383	28.2670	27,3234	27.2658	26,7096	26.6580

n	8	9	10			
Z						
2	1.0209	1.0161	1,0166	1.0130	1.0135	1.0107
3	2.3227	2.3135	2.3077	2.3007	2.2968	2.2914
4	4.1558	4.1421	4.1234	4.1130	4.1001	4.0921
5	6.5201	6.5020	6.4637	6.4501	6.4234	6.4128
6	9.4156	9.3931	9.3288	9.3118	9.2666	9.2534
7	12.8424	12.8154	12.7186	12.6983	12.6299	12.6141
8	16.8004	16.7690	16.6331	16.6094	16.5132	16.4948
9	21.2897	21.2539	21.0722	21.0452	20.9165	20.8955
10	26.3103	26.2700	26.0361	26.0057	25.8398	25.8182

${}_2(0,1)_n^- {}^3P^0$						
n	5	6	7			
Z	Present results	Dieng <i>et al.</i> (2014)	Present results	Dieng <i>et al.</i> (2014)	Present results	Dieng <i>et al.</i> (2014)
2	1.0587	1.0582	1.0405	1.0397	1.0296	1.0282
3	2.4457	2.4446	2.3853	2.3839	2.3491	2.3467
4	4.4126	4.4111	4.2856	4.2836	4.2094	4.2059
5	6.9596	6.9576	6.7416	6.7389	6.6106	6.6060
6	10.0865	10.0840	9.7531	9.7498	9.5525	9.5468
7	13.7935	13.7905	13.3201	13.316	13.0353	13.0285
8	18.0804	18.0770	17.4427	17.4381	17.0589	17.0510
9	22.9474	22.9435	22.1208	22.1157	21.6234	21.6144
10	28.3943	28.3899	27.3546	27.3487	26.7286	26.7185
n	8		9		10	
Z						
2	1.0225	1.0216	1.0178	1.0173	1.0143	1.0122
3	2.3257	2.3240	2.3097	2.3088	2.2983	2.2944
4	4.1601	4.1576	4.1263	4.1250	4.1023	4.0965
5	6.5257	6.5224	6.4677	6.4660	6.4262	6.4186
6	9.4227	9.4186	9.3337	9.3316	9.2702	9.2608
7	12.8508	12.8459	12.7244	12.7219	12.6341	12.6229
8	16.8102	16.8045	16.6398	16.6369	16.5181	16.5050
9	21.3009	21.2944	21.0799	21.0766	20.9220	20.9071
10	26.3228	26.3155	26.0447	26.0409	25.8460	25.8293

**Table 15.** Comparison of energy resonances ( $-E$ , Ryb) of doubly excited  ${}^A_3(1,1)_n^{1,3}P^0$  ( $n = 5 - 10$ ) states of helium-like ions ( $Z = 2 - 10$ ) with Dieng et al. (2014) theoretical results.

${}^A_3(1,1)_n^{1}P^0$						
Z	5		6		7	
	Present results	Dieng et al. (2014)	Present results	Dieng et al. (2014)	Present results	Dieng et al. (2014)
2	0.5057	0.4990	0.4863	0.4815	0.4748	0.4706
3	1.2002	1.1880	1.1378	1.1289	1.1006	1.0928
4	1.9393	1.9308	1.9051	1.8991	1.8808	1.8763
5	3.4959	3.4727	3.2741	3.2572	3.1413	3.1263
6	4.4250	4.4111	4.3353	4.3254	4.4250	4.4111
7	7.0006	6.9662	6.5216	6.4965	6.2341	6.2119
8	9.2062	9.1663	8.5620	8.5329	8.1750	8.1493
9	11.7141	11.6687	10.8801	10.8470	10.3790	10.3497
10	14.5242	14.4732	13.4761	13.4389	12.8461	12.8132
${}^A_3(1,1)_n^{3}P^0$						
Z	5		6		7	
	Present results	Dieng et al. (2014)	Present results	Dieng et al. (2014)	Present results	Dieng et al. (2014)
2	0.5135	0.5101	0.4905	0.4888	0.4774	0.4758
3	1.2141	1.2082	1.1454	1.1423	1.1053	1.1024
4	2.2170	2.2085	2.0781	2.0736	1.9962	1.9920
5	3.5220	3.5110	3.2886	3.2827	3.1501	3.1446
6	5.1293	5.1158	4.7768	4.7695	4.5670	4.5603
7	7.0389	7.0227	6.5428	6.5341	6.2470	6.2391
8	9.2506	9.2319	8.5866	8.5765	8.1901	8.1808
9	11.7646	11.7433	10.9081	10.8967	10.3962	10.3856
10	14.5808	14.5570	13.5075	13.4947	12.8653	12.8535
${}^A_3(1,1)_n^{1}P^0$						
Z	8		9		10	
	Present results	Dieng et al. (2014)	Present results	Dieng et al. (2014)	Present results	Dieng et al. (2014)
2	0.4692	0.4682	0.4637	0.4633	0.4599	0.4598
3	1.0797	1.0780	1.0625	1.0617	1.0503	1.0501
4	1.9437	1.9412	1.9082	1.9071	1.8829	1.8827
5	3.0612	3.0579	3.0008	2.9993	2.9578	2.9574
6	4.4321	4.4280	4.3403	4.3385	4.2749	4.2744
7	6.0566	6.0517	5.9267	5.9246	5.8342	5.8336
8	7.9345	7.9288	7.7600	7.7575	7.6357	7.6351
9	10.0658	10.0593	9.8403	9.8375	9.6795	9.6787
10	12.4506	12.4433	12.1674	12.1643	11.9654	11.9646

**Table 16.** Comparison of energy resonances ( $-E$ , Ryb) of doubly excited  ${}^A_4(2,1)_n^{1,3}P^0$  ( $n = 5 - 10$ ) states of helium-like ions ( $Z = 2 - 10$ ) with Dieng et al. (2014) theoretical results.

${}^A_4(2,1)_n^{1}P^0$						
Z	5		6		7	
	Present results	Dieng et al. (2014)	Present results	Dieng et al. (2014)	Present results	Dieng et al. (2014)
2	0.3217	0.3107	0.2976	0.2914	0.2839	0.2794
3	0.7813	0.7618	0.7105	0.6995	0.6693	0.6613
4	1.4459	1.4179	1.3040	1.2881	1.2206	1.2091
5	2.3156	2.2790	2.0781	2.0573	1.9378	1.9226
6	3.3902	3.3450	3.0328	3.0071	2.8207	2.8020
7	4.6698	4.6161	4.1680	4.1374	3.8694	3.8472
8	6.1544	6.0922	5.4837	5.4482	5.0840	5.0582
9	7.8440	7.7733	6.9800	6.9396	6.4644	6.4350
10	9.7386	9.6594	8.6569	8.6116	8.0106	7.9776

<b>n</b>	<b>8</b>	<b>9</b>	<b>10</b>	
<b>Z</b>				
2	0.2753	0.2724	0.2697	0.2677
3	0.6433	0.6379	0.6257	0.6222
4	1.1674	1.1596	1.1314	1.1263
5	1.8479	1.8376	1.7868	1.7802
6	2.6845	2.6790	2.5919	2.5837
7	3.6775	3.6624	3.5467	3.5369
8	4.8266	4.8091	4.6512	4.6398
9	6.1321	6.1121	5.9054	5.8924
10	7.5937	7.5714	7.3092	7.2948

${}^A(2,1)_n^- {}^3P^o$						
<b>n</b>	<b>5</b>	<b>6</b>	<b>7</b>			
<b>Z</b>	<b>Present results</b>	<b>Dieng <i>et al.</i> (2014)</b>	<b>Present results</b>	<b>Dieng <i>et al.</i> (2014)</b>	<b>Present results</b>	<b>Dieng <i>et al.</i> (2014)</b>
2	0.3301	0.3197	0.3021	0.2971	0.2866	0.2835
3	0.7958	0.7778	0.7185	0.7098	0.6741	0.6686
4	1.4666	1.4408	1.3154	1.3030	1.2275	1.2196
5	1.9467	1.9364	1.8537	1.8478	1.7909	1.7879
6	3.4230	3.3820	3.0509	3.0311	2.8317	2.8190
7	4.7087	4.6601	4.1894	4.1660	3.8825	3.8675
8	6.1994	6.1432	5.5086	5.4814	5.0992	5.0817
9	7.8951	7.8312	7.0083	6.9774	6.4816	6.4618
10	9.7959	9.7243	8.6885	8.6539	8.0299	8.0076

<b>n</b>	<b>8</b>	<b>9</b>	<b>10</b>	
<b>Z</b>				
2	0.2771	0.2753	0.2709	0.2700
3	0.6464	0.6432	0.6279	0.6263
4	1.1719	1.1674	1.1345	1.1322
5	1.7464	1.7451	1.9467	1.9364
6	2.6918	2.6844	2.5969	2.5932
7	3.6860	3.6773	3.5526	3.5483
8	4.8366	4.8265	4.6580	4.6530
9	6.1434	6.1319	5.9132	5.9074
10	7.6064	7.5935	7.3180	7.3115

**Table 17.** Comparison of energy resonances ( $-E$ , Ryb) of doubly excited  ${}_5(3,1)_n^A {}^{1,3}P^o$  ( $n = 6 - 10$ ) states of helium-like ions ( $Z = 2 - 10$ ) with Dieng *et al.* (2014) theoretical results.

${}_5(3,1)_n^+ {}^1P^o$						
<b>n</b>	<b>6</b>	<b>7</b>	<b>8</b>			
<b>Z</b>	<b>Present results</b>	<b>Dieng <i>et al.</i> (2014)</b>	<b>Present results</b>	<b>Dieng <i>et al.</i> (2014)</b>	<b>Present results</b>	<b>Dieng <i>et al.</i> (2014)</b>
2	0.2137	0.1992	0.1975	0.1887	0.1877	0.1822
3	0.5186	0.4930	0.4733	0.45753	0.4449	0.4351
4	0.9592	0.9223	0.8698	0.8472	0.8134	0.7993
5	1.5353	1.4872	1.3872	1.3576	1.2932	1.2747
6	2.2469	2.1876	2.0254	1.9889	1.8842	1.8613
7	3.0942	3.0236	2.7844	2.7410	2.5864	2.5592
8	4.0769	3.9952	3.6643	3.6139	3.3999	3.3683
9	5.1952	5.1023	4.6649	4.6076	4.3246	4.2887
10	6.4491	6.3449	5.7864	5.7222	5.3606	5.3204

<b>n</b>	<b>9</b>		<b>10</b>	
<b>Z</b>	<b>Present results</b>	<b>Dieng <i>et al.</i> (2014)</b>	<b>Present results</b>	<b>Dieng <i>et al.</i> (2014)</b>
2	0.1813	0.1779	0.1769	0.1747
3	0.4261	0.4199	0.4129	0.4089
4	0.7755	0.7666	0.7489	0.7431
5	1.2297	1.2181	1.1848	1.1773
6	1.7886	1.7742	1.7208	1.7116
7	2.4521	2.4350	2.3568	2.3458
8	3.2203	3.2005	3.0928	3.0800
9	4.0933	4.0707	3.9288	3.9142
10	5.0709	5.0456	4.8648	4.8484

${}^3S_1$						
$n = 6$		$n = 7$		$n = 8$		
Z	Present results	Dieng et al. (2014)	Present results	Dieng et al. (2014)	Present results	Dieng et al. (2014)
2	0.2185	0.2042	0.2004	0.1921	0.1896	0.1847
3	0.5268	0.5021	0.4782	0.4636	0.4482	0.4396
4	0.9708	0.9355	0.8769	0.8562	0.8180	0.8057
5	1.5503	1.5045	1.3963	1.3694	1.2991	1.2831
6	2.2653	2.2090	2.0366	2.0035	1.8915	1.8718
7	3.1159	3.0491	2.7977	2.7584	2.5951	2.5717
8	4.1021	4.0247	3.6796	3.6341	3.4099	3.3828
9	5.2238	5.1359	4.6823	4.6306	4.3360	4.3052
10	6.4811	6.3827	5.8059	5.7479	5.3734	5.3389

  

${}^3P_0$				
$n = 9$		$n = 10$		
Z	Present results	Dieng et al. (2014)	Present results	Dieng et al. (2014)
2	0.1826	0.1797	0.1778	0.1761
3	0.4283	0.4232	0.4144	0.4115
4	0.7787	0.7715	0.7511	0.7469
5	1.2338	1.2244	1.1878	1.1822
6	1.7936	1.7820	1.7244	1.7176
7	2.4581	2.4443	2.3611	2.3530
8	3.2272	3.2113	3.0978	3.0884
9	4.1011	4.0830	3.9344	3.9238
10	5.0797	5.0594	4.8711	4.8591

**Table 18.** Comparison of energy resonances ( $-E$ , Ryb) of doubly excited  ${}^3P_0$  ( $n = 7 - 10$ ) states of helium-like ions ( $Z = 2 - 10$ ) with Dieng et al. (2014) theoretical results.

${}^3P_1$				
$n = 7$		$n = 8$		
Z	Present results	Dieng et al. (2014)	Present results	Dieng et al. (2014)
2	0.1525	0.1405	0.1413	0.1339
3	0.3699	0.3487	0.3392	0.3262
4	0.5842	0.5725	0.5563	0.5485
5	1.0938	1.0543	0.9955	0.9712
6	1.6003	1.5516	1.4539	1.4239
7	2.2032	2.1453	1.9991	1.9634
8	2.9025	2.8354	2.6310	2.5897
9	3.6981	3.6219	3.3498	3.3028
10	4.5901	4.5047	4.1554	4.1027

  

${}^3P_2$				
$n = 9$		$n = 10$		
Z	Present results	Dieng et al. (2014)	Present results	Dieng et al. (2014)
2	0.1341	0.1295	0.1292	0.1262
3	0.3190	0.3109	0.3050	0.2997
4	0.5842	0.5725	0.5563	0.5485
5	0.9296	0.9143	0.8832	0.8733
6	1.3553	1.3364	1.2856	1.2735
7	1.8612	1.8388	1.7637	1.7492
8	2.4473	2.4214	2.3172	2.3005
9	3.1137	3.0842	2.9463	2.9273
10	3.8604	3.8273	3.6510	3.6296

${}^3D_1$				
$n = 7$		$n = 8$		
Z	Present results	Dieng et al. (2014)	Present results	Dieng et al. (2014)
2	0.1555	0.1434	0.1432	0.1360
3	0.3750	0.3541	0.3425	0.3300
4	0.6908	0.6611	0.6286	0.6108
5	1.1030	1.0645	1.0016	0.9784
6	1.6116	1.5643	1.4613	1.4328
7	2.2166	2.1604	2.0078	1.9740
8	2.9180	2.8529	2.6411	2.6020
9	3.7157	3.6418	3.3613	3.3168
10	4.6098	4.5270	4.1682	4.1184

n	9		10		
	Z	Present results	Dieng <i>et al.</i> (2014)	Present results	Dieng <i>et al.</i> (2014)
2	0.1354	0.1311	0.1301	0.1274	
3	0.3213	0.3137	0.3066	0.3019	
4	0.5874	0.5765	0.5586	0.5518	
5	0.9337	0.9196	0.8862	0.8774	
6	1.3603	1.3430	1.2893	1.2785	
7	1.8672	1.8466	1.7680	1.7552	
8	2.4543	2.4305	2.3222	2.3074	
9	3.1216	3.0946	2.9520	2.9352	
10	3.8692	3.8389	3.6573	3.6385	

**Table 19.** Comparison of energy resonances ( $-E$ , Ryb) of doubly excited  $\gamma((5,1)_n^A \text{ } ^{1,3}P^0)$ ,  $s(6,1)_n^A \text{ } ^{1,3}P^0$  and  $\delta(7,1)_{10}^A \text{ } ^{1,3}P^0$  states of helium-like ( $Z = 2-10$ ) with Dieng *et al.* (2014) theoretical results.

$\gamma((5,1)_n^+ \text{ } ^1\text{P}^0)$						
n	8		9		10	
	Z	Present results	Dieng <i>et al.</i> (2014)	Present results	Dieng <i>et al.</i> (2014)	Present results
2	0.1144	0.1045	0.1064	0.1002	0.1009	0.0969
3	0.2773	0.2600	0.2557	0.2448	0.2408	0.2337
4	0.5123	0.4876	0.4705	0.4549	0.4414	0.4313
5	0.8193	0.7872	0.7509	0.7305	0.7029	0.6897
6	1.1984	1.1589	1.0967	1.0717	1.0252	1.0089
7	1.6496	1.6027	1.5081	1.4784	1.4083	1.3889
8	2.1729	2.1185	1.9850	1.9505	1.8522	1.8297
9	2.7682	2.7064	2.5273	2.4882	2.3569	2.3314
10	3.4356	3.3664	3.1352	3.0914	2.9225	2.8939

$s(6,1)_9^+ \text{ } ^1\text{P}^0$						
$s(6,1)_{10}^+ \text{ } ^1\text{P}^0$		$\delta(6,1)_{10}^+ \text{ } ^1\text{P}^0$				
Z	n	Present results	Dieng <i>et al.</i> (2014)	Present results	Dieng <i>et al.</i> (2014)	
2	0.0890	0.0812	0.0831	0.0779	0.0713	0.0650
3	0.2157	0.2020	0.1999	0.1909	0.1726	0.1617
4	0.3983	0.3788	0.3680	0.3551	0.3186	0.3031
5	0.6369	0.6115	0.5873	0.5706	0.5094	0.4891
6	0.9314	0.9002	0.8579	0.8373	0.7448	0.7199
7	1.2818	1.2448	1.1797	1.1553	1.0249	0.9954
8	1.6882	1.6453	1.5528	1.5245	1.3497	1.3155
9	2.1505	2.1018	1.9771	1.9450	1.7191	1.6804
10	2.6688	2.6142	2.4527	2.4167	2.1333	2.090

$\gamma((5,1)_n^- \text{ } ^3\text{P}^0)$						
n	7		8		10	
	Z	Present results	Dieng <i>et al.</i> (2014)	Present results	Dieng <i>et al.</i> (2014)	Present results
2	0.1164	0.1064	0.1077	0.1015	0.1019	0.0980
3	0.2807	0.2634	0.2580	0.2472	0.2424	0.2356
4	0.5170	0.4925	0.4738	0.4585	0.4437	0.4340
5	0.8254	0.7936	0.7550	0.7352	0.7059	0.6932
6	1.2059	1.1668	1.1018	1.0775	1.0289	1.0132
7	1.6584	1.6121	1.5141	1.4852	1.4126	1.3941
8	2.1830	2.1295	1.9920	1.9585	1.8572	1.8358
9	2.7797	2.7189	2.5353	2.4973	2.3626	2.3383
10	3.4485	3.3804	3.1441	3.1015	2.9289	2.9017

$s(6,1)_9^- \text{ } ^3\text{P}^0$						
$s(6,1)_{10}^- \text{ } ^3\text{P}^0$		$\delta(6,1)_{10}^- \text{ } ^3\text{P}^0$				
Z	n	Present results	Dieng <i>et al.</i> (2014)	Present results	Dieng <i>et al.</i> (2014)	
2	0.0904	0.0824	0.0841	0.0789	0.0723	0.0658
3	0.2180	0.2042	0.2016	0.1926	0.1743	0.1632
4	0.4016	0.3820	0.3703	0.3575	0.3210	0.30522
5	0.6411	0.6157	0.5903	0.5738	0.5124	0.4920
6	0.9365	0.9054	0.8616	0.8412	0.7485	0.7234
7	1.2879	1.2509	1.1841	1.1599	1.0293	0.9995
8	1.6952	1.6525	1.5578	1.5299	1.3547	1.3205
9	2.1585	2.1099	1.9828	1.9511	1.7249	1.6859
10	2.6777	2.6234	2.4591	2.4236	2.1397	2.0961

**Table 20.** Comparison of excitation energy of some states of He I with some experiments. Both resonance positions are expressed in eV and the energies are measured from the ground state of He I ( $E = -79,01$  eV). The infinite Rydberg unit (1 Ry = 13.60569 eV) is used for energy conversion.

$n(K, T)_n^A$	$Nlnl' 2S+I L''$	Present results	Dieng et al. (2014)	Domke et al. (1996)	Hicks and Comer (1975)	Dhez and Ederer (1973)	Madden and Codling (1963, 1965)
${}_2(0,1)_3^+$	$2s3p\ ^1P^0$	63.626	63.710	63.658	63.650	63.650	63.655
${}_2(0,1)_4^+$	$2s4p\ ^1P^0$	64.340	64.191	64.467	64.450	64.470	64.466
${}_2(0,1)_5^+$	$2s5p\ ^1P^0$	64.703	64.818	64.816			64.816
${}_2(0,1)_6^+$	$2s6p\ ^1P^0$	64.908	65.002				64.999
${}_2(0,1)_7^+$	$2s7p\ ^1P^0$	65.036	65.119				
${}_2(0,1)_8^+$	$2s8p\ ^1P^0$	65.120	65.185				
${}_2(0,1)_9^+$	$2s9p\ ^1P^0$	65.178	65.227				
${}_2(0,1)_{10}^+$	$2s10p\ ^1P^0$	65.220	65.259				
${}_3(1,1)_4^+$	$3s4p\ ^1P^0$	71.625	71.535	71.623		71.660	69,900
${}_3(1,1)_5^+$	$3s5p\ ^1P^0$	72.130	72.220	72.179		72.200	
${}_3(1,1)_6^+$	$3s6p\ ^1P^0$	72.394	72.459				
${}_3(1,1)_7^+$	$3s7p\ ^1P^0$	72.550	72.607				
${}_3(1,1)_8^+$	$3s8p\ ^1P^0$	72.649	72.692				
${}_3(1,1)_9^+$	$3s9p\ ^1P^0$	72.716	72.747				
${}_3(1,1)_{10}^+$	$3s10p\ ^1P^0$	72.764	72.787				

$n(K, T)_n^A$	$Nlnl' 2S+I L''$	Present results	Dieng et al. (2014)	Shulz et al. (1996)	Hicks and Comer (1975)	Rudd (1964, 1965)
${}_2(0,1)_3^-$	$2s3p\ ^3P^0$	63.090	63.054	62.761	63.070	63.080
${}_2(0,1)_4^-$	$2s4p\ ^3P^0$	64.138	63.871	64.613	64.230	64.220
${}_2(0,1)_5^-$	$2s5p\ ^3P^0$	64.605	64.136	64.659	64.690	64.710
${}_2(0,1)_6^-$	$2s6p\ ^3P^0$	64.854	64.864			
${}_2(0,1)_7^-$	$2s7p\ ^3P^0$	65.002	65.020			
${}_2(0,1)_8^-$	$2s8p\ ^3P^0$	65.098	65.110			
${}_2(0,1)_9^-$	$2s9p\ ^3P^0$	65.163	65.169			
${}_2(0,1)_{10}^-$	$2s10p\ ^3P^0$	65.209	65.238			
${}_3(1,1)_4^-$	$3s4p\ ^3P^0$	71.400	71.387			
${}_3(1,1)_5^-$	$3s5p\ ^3P^0$	72.024	72.069			
${}_3(1,1)_6^-$	$3s6p\ ^3P^0$	72.336	72.360			
${}_3(1,1)_7^-$	$3s7p\ ^3P^0$	72.514	72.781			
${}_3(1,1)_8^-$	$3s8p\ ^3P^0$	72.626	72.639			
${}_3(1,1)_9^-$	$3s9p\ ^3P^0$	72.701	72.706			
${}_3(1,1)_{10}^-$	$3s10p\ ^3P^0$	72.753	72.754			

**Table 21.** Comparison of excitation energy of some states of Li II. Both resonance positions are expressed in eV and the energies are measured from the ground state of Li II ( $E = -198.09$  eV). The infinite Rydberg unit (1 Ry = 13.60569 eV) is used for energy conversion.

$n(K, T)_n^A$	$Nlnl' 2S+I L''$	Present results	Dieng et al. (2014)	Diehl et al. (1999)	Scully et al. (2006)
${}_2(0,1)_3^+$	$2s3p\ ^1P^0$	160.907	161.071	161.070	161.110
${}_2(0,1)_4^+$	$2s4p\ ^1P^0$	163.660	163.382	163.950	164.000
${}_2(0,1)_5^+$	$2s5p\ ^1P^0$	164.995	165.218	165.270	165.290
${}_2(0,1)_6^+$	$2s6p\ ^1P^0$	165.737	165.917	165.920	165.980
${}_2(0,1)_7^+$	$2s7p\ ^1P^0$	166.191	166.351		166.400
${}_2(0,1)_8^+$	$2s8p\ ^1P^0$	166.488	166.613		
${}_2(0,1)_9^+$	$2s9p\ ^1P^0$	166.693	166.788		
${}_2(0,1)_{10}^+$	$2s10p\ ^1P^0$	166.840	166.914		
${}_3(1,1)_4^+$	$3s4p\ ^1P^0$	180.163			
${}_3(1,1)_5^+$	$3s5p\ ^1P^0$	181.760			
${}_3(1,1)_6^+$	$3s6p\ ^1P^0$	182.610			
${}_3(1,1)_7^+$	$3s7p\ ^1P^0$	183.115			
${}_3(1,1)_8^+$	$3s8p\ ^1P^0$	183.441			
${}_3(1,1)_9^+$	$3s9p\ ^1P^0$	183.663			
${}_3(1,1)_{10}^+$	$3s10p\ ^1P^0$	183.820			

$n(\text{K}, \text{T})_n^A$	$Nlml'{}^{2S+I}L''$	Present results	Dieng <i>et al.</i> (2014)	Diehl <i>et al.</i> (1999)
${}_2(0,1)_3^-$	$2s3p\ {}^3\text{P}^0$	159.910	159.845	
${}_2(0,1)_4^-$	$2s4p\ {}^3\text{P}^0$	163.285	162.809	
${}_2(0,1)_5^-$	$2s5p\ {}^3\text{P}^0$	164.815	164.829	159.87
${}_2(0,1)_6^-$	$2s6p\ {}^3\text{P}^0$	165.637	165.656	163.45
${}_2(0,1)_7^-$	$2s7p\ {}^3\text{P}^0$	166.129	166.162	
${}_2(0,1)_8^-$	$2s8p\ {}^3\text{P}^0$	166.447	166.471	
${}_2(0,1)_9^-$	$2s9p\ {}^3\text{P}^0$	166.665	166.677	
${}_2(0,1)_{10}^-$	$2s10p\ {}^3\text{P}^0$	166.820	166.873	
${}_3(1,1)_4^-$	$3s4p\ {}^3\text{P}^0$	179.6278	179.587	
${}_3(1,1)_5^-$	$3s5p\ {}^3\text{P}^0$	181.5070	181.651	
${}_3(1,1)_6^-$	$3s6p\ {}^3\text{P}^0$	182.4703	182.548	
${}_3(1,1)_7^-$	$3s7p\ {}^3\text{P}^0$	183.0306	183.091	
${}_3(1,1)_8^-$	$3s8p\ {}^3\text{P}^0$	183.3854	183.423	
${}_3(1,1)_9^-$	$3s9p\ {}^3\text{P}^0$	183.6243	183.644	
${}_3(1,1)_{10}^-$	$3s10p\ {}^3\text{P}^0$	183.7929	183.802	

**Table 22.** Comparison of resonance energies ( $-E$ , a.u) of doubly excited  ${}_3(1,1)_n^+ {}^1\text{P}^0$  ( $n = 4 - 10$ ) states of helium-like ions ( $Z = 2 - 40$ ) with the theoretical results Sakho (2017) and Bachau *et al.* (1991).

${}_3(1,1)_n^+ {}^1\text{P}^0$							
n	4		5		6		Sakho (2017)
	Z	Present results	Sakho (2017)	Bachau <i>et al.</i> (1991)	Present results	Sakho (2017)	Present results
2	0.271	0.272	0.271	0.253	0.251	0.243	0.241
3	0.659	0.659	0.660	0.600	0.597	0.569	0.566
4	1.220	1.221	1.222	1.098	1.095	1034	1.029
5	1.954	1.956	1.957	1.748	1.743	1.637	1.631
6	2.863	2.864	2.865	2.549	2.542	2.379	2.372
7	3.945	3.946	3.948	3.500	3.493	3.261	3.252
8	5.200	5.202	5.204	4.603	4.595	4.281	4.270
9	6.629	6.631	6.633	5.857	5.847	5.440	5.428
10	8.232	8.234	8.236	7.262	7.251	6.738	6.724
11	10.008	10.011		8.818	8.806	8.175	8.160
12	11.958	11.961		10.526	10.512	9.751	9.734
13	14.082	14.085		12.384	12.370	11.465	11.447
14	16.379	16.382		14.393	14.378	13.319	13.299
15	18.849	18.853		16.554	16.537	15.311	15.290
16	21.494	21.498		18.866	18.848	17.443	17.420
17	24.312	24.316		21.329	21.309	19.713	19.689
18	27.303	27.308		23.942	23.922	22.122	22.096
19	30.469	30.474		26.708	26.686	24.670	24.643
20	33.807	33.813		29.624	29.601	27.357	27.328
21	37.320	37.325		32.691	32.667	30.182	30.153
22	41.006	41.012		35.909	35.884	33.147	33.116
23	44.865	44.872		39.279	39.253	36.251	36.218
24	48.899	48.905		42.799	42.772	39.493	39.459
25	53.106	53.112		46.471	46.443	42.874	42.839
26	57.486	57.493		50.294	50.264	46.395	46.357
27	62.040	62.047		54.268	54.237	50.054	50.015
28	66.768	66.775		58.393	58.361	53.852	53.811
29	71.669	71.677		62.669	62.636	57.788	57.747
30	76.744	76.752		67.096	67.062	61.864	61.821
31	81.993	82.001		71.675	71.639	66.079	66.034
32	87.415	87.423		76.404	76.368	70.432	70.386
33	93.013	93.019		81.285	81.247	74.925	74.877
34	98.780	98.789		86.317	86.277	79.556	79.507
35	104.723	104.732		91.499	91.459	84.326	84.276
36	110.839	110.849		96.833	96.792	89.235	89.183
37	117.130	117.140		102.318	102.276	94.283	94.230
38	123.593	123.604		107.954	107.911	99.470	99.415
39	130.231	130.241		113.742	113.697	104.796	104.739
40	137.042	137.053		119.680	119.634	110.261	110.203
41	144.026			125.770		115.864	
42	151.185			132.010		121.607	
43	158.517			138.402		127.488	
44	166.022			144.945		133.508	
45	173.701			151.639		139.667	

${}^3(1,1)_n^+ {}^1P^o$						
n	7		8		9	
	Z	Present results	Sakho (2017)	Present results	Sakho (2017)	Present results
2	0.237	0.236	0.234	0.232	0.231	0.230
3	0.550	0.547	0.538	0.535	0.530	0.524
4	0.995	0.991	0.970	0.965	0.953	0.940
5	1.571	1.565	1.528	1.522	1.498	1.477
6	2.278	2.271	2.213	2.206	2.168	2.136
7	3.117	3.109	3.024	3.016	2.960	2.915
8	4.088	4078	3.962	3.953	3.877	3.815
9	5.190	5.179	5.027	5.017	4.916	4.837
10	6.423	6.411	6.219	6.209	6.079	5.980
11	7.788	7.775	7.538	7.524	7.366	7.243
12	9.285	9.275	8.983	8.968	8.776	8.628
13	10.913	10.897	10.555	10.539	10.309	10.134
14	12.672	12.655	12.253	12.236	11.966	11.761
15	14.563	14.545	14.079	14.060	13.747	13.509
16	16.586	16.566	16.031	16.011	15.651	15.379
17	18.740	18.719	18.110	18.089	17.678	17.369
18	21.026	21.004	20.315	20.293	19.829	19.481
19	23.443	23.420	22.648	22.624	22.103	21.713
20	25.992	25.967	25.107	25.082	24.501	24.067
21	28.672	28.646	27.693	27.666	27.022	26.542
22	31.484	31.456	30.405	30.377	29.666	29.138
23	34.427	34.398	33.245	33.215	32.434	31.855
24	37.502	37.472	36.211	36.180	35.326	34.693
25	40.708	40.677	39.303	39.272	38.341	37.653
26	44.046	44.013	42.523	42.490	41.479	40.733
27	47.515	47.482	45.869	45.835	44.741	43.935
28	51.116	51.081	49.342	49.306	48.126	47.257
29	54.849	54.812	52.942	52.905	51.635	50.701
30	58.713	58.675	56.668	56.630	55.267	54.266
31	62.708	62.669	60.521	60.482	59.023	57.952
32	66.835	66.795	64.501	64.460	62.902	61.759
33	71.093	71.052	68.608	68.566	66.905	65.687
34	75.483	75.440	72.841	72.798	71.031	69.736
35	80.005	79.961	77.202	77.157	75.281	73.907
36	84.658	84.612	81.689	81.642	79.654	78.198
37	89.442	89.396	86.302	86.255	84.150	82.611
38	94.358	94.310	91.042	90.994	88.770	87.145
39	99.406	99.357	95.910	95.859	93.513	91.800
40	104.585	104.535	100.903	100.852	98.380	96.576
41	109.896		106.024		103.370	101.473
42	115.338		111.271		108.484	106.491
43	120.912		116.645		113.721	111.630
44	126.617		122.146		119.082	116.891
45	132.453		127.773		124.566	122.272

**Table 23.** Comparison of energy positions ( $-E$ , a.u) of doubly excited  ${}^3(1,1)_n^- {}^3P^o$  ( $n = 4 - 10$ ) states of helium-like ions ( $Z = 2 - 40$ ) with the theoretical results Sakho (2017) and Bachau et al. (1991).

${}^3(1,1)_n^- {}^3P^o$						
n	4		5		6	
	Z	Present results	Sakho (2017)	Bachau et al. (1991)	Present results	Sakho (2017)
2	0.280	0.281	0.279	0.257	0.255	0.245
3	0.673	0.675	0.675	0.607	0.604	0.573
4	1.241	1.244	1.245	1.108	1.103	1.039
5	1.982	1.985	1.957	1.761	1.754	1.644
6	2.896	2.901	2.906	2.565	2.556	2.388
7	3.985	3.990	3.997	3.519	3.510	3.271
8	5.246	5.253	5.261	4.625	4.614	4.293
9	6.682	6.689	6.699	5.882	5.869	5.454
10	8.291	8.299	8.310	7.290	7.276	6.754
11	10.074	10.082		8.850	8.834	8.192
12	12.030	12.039		10.560	10.542	9.770
13	14.160	14.170		12.421	12.402	11.486
14	16.463	16.474		14.434	14.413	13.341
15	18.940	18.952		16.598	16.575	15.335
16	21.591	21.604		18.912	18.889	17.468
17	24.415	24.429		21.378	21.353	19.740
18	27.413	27.428		23.995	23.968	22.151
19	30.585	30.600		26.763	26.735	24.701
20	33.930	33.946		29.682	29.653	27.389
21	37.449	37.466		32.753	32.721	30.217

$^3(\mathbf{1}, \mathbf{1})_n^- \text{ } ^3\text{P}^o$								
n	4		5		6			
	Z	Present results	Sakho (2017)	Bachau et al. (1991)	Present results	Sakho (2017)	Present results	Sakho (2017)
22	41.141	41.159			35.974	35.941	33.183	33.144
23	45.007	45.026			39.347	39.312	36.288	36.247
24	49.047	49.066			42.870	42.834	39.532	39.489
25	53.260	53.280			46.545	46.508	42.915	42.870
26	57.647	57.668			50.371	50.332	46.437	46.390
27	62.207	62.229			54.348	54.307	50.098	50.049
28	66.941	66.964			58.476	58.434	53.898	53.847
29	71.849	71.872			62.755	62.712	57.836	57.784
30	76.930	76.954			67.186	67.140	61.914	61.859
31	82.185	82.210			71.767	71.720	66.130	66.074
32	87.614	87.639			76.500	76.451	70.485	70.427
33	93.216	93.242			81.383	81.333	74.979	74.920
34	98.991	99.019			86.418	86.367	79.612	79.551
35	104.941	104.969			91.604	91.551	84.384	84.321
36	111.064	111.093			96.941	96.886	89.295	89.230
37	117.360	117.390			102.429	102.373	94.345	94.277
38	123.830	123.861			108.068	108.011	99.533	99.464
39	130.474	130.506			113.859	113.799	104.861	104.790
40	137.292	137.324			119.800	119.739	110.327	110.254
41	144.283				125.892		115.938	
42	151.447				132.136		121.676	
43	158.785				138.531		127.559	
44	166.297				145.077		133.581	
45	173.983				151.774		139.742	

$^3(\mathbf{1}, \mathbf{1})_n^- \text{ } ^3\text{P}^o$					
n	7		8		
	Z	Present results	Sakho (2017)	Present results	Sakho (2017)
2		0.239	0.237	0.235	0.233
3		0.553	0.549	0.540	0.536
4		0.998	0.992	0.972	0.967
5		1.575	1.568	1.531	1.524
6		2.284	2.274	2.216	2.208
7		3.124	3.113	3.028	3.019
8		4.095	4.082	3.967	3.956
9		5.198	5.184	5.033	5.020
10		6.433	6.417	6.225	6.211
11		7.799	7.781	7.544	7.529
12		9.296	9.277	8.990	8.973
13		10.925	10.904	10.563	10.544
14		12.686	12.663	12.262	12.242
15		14.578	14.553	14.088	14.067
16		16.602	16.575	16.041	16.018
17		18.757	18.729	18.121	18.096
18		21.044	21.014	20.327	20.301
19		23.462	23.430	22.660	22.632
20		26.012	25.978	25.120	25.090
21		28.693	28.658	27.706	27.675
22		31.506	31.469	30.420	30.387
23		34.450	34.411	33.260	33.225
24		37.526	37.485	36.226	36.191
25		40.733	40.691	39.320	39.283
26		44.072	44.028	42.540	42.501
27		47.543	47.497	45.887	45.847
28		51.144	51.097	49.361	49.319
29		54.878	54.829	52.961	52.918
30		58.743	58.692	56.688	56.643
31		62.739	62.687	60.542	60.496
32		66.867	66.813	64.523	64.475
33		71.127	71.071	68.630	68.580
34		75.518	75.460	72.864	72.813
35		80.040	79.981	77.225	77.172
36		84.694	84.633	81.713	81.658
37		89.480	89.417	86.327	86.271
38		94.397	94.332	91.068	91.010
39		99.446	99.379	95.936	95.877
40		104.626	104.557	100.930	100.870
41		109.938		106.051	
42		115.381		111.299	
43		120.955		116.674	

$^3(\mathbf{1}, \mathbf{1})_n^- \ ^3\text{P}^0$			
n	Z	7	8
		Present results	Sakho (2017)
44		126.662	122.175
45		132.499	127.804

  

$^3(\mathbf{1}, \mathbf{1})_n^- \ ^3\text{P}^0$			
n	Z	9	10
		Present results	Sakho (2017)
2		0.232	0.230
3		0.531	0.528
4		0.954	0.950
5		1.500	1.495
6		2.170	2.163
7		2.963	2.955
8		3.880	3.870
9		4.920	4.909
10		6.084	6.071
11		7.371	7.357
12		8.781	8.766
13		10.315	10.299
14		11.973	11.955
15		13.753	13.734
16		15.658	15.637
17		17.686	17.664
18		19.837	19.814
19		22.111	22.087
20		24.510	24.484
21		27.031	27.004
22		29.676	29.648
23		32.445	32.415
24		35.337	35.306
25		38.352	38.320
26		41.491	41.457
27		44.753	44.718
28		48.139	48.103
29		51.649	51.611
30		55.281	55.242
31		59.037	58.997
32		62.917	62.875
33		66.920	66.877
34		71.047	71.002
35		75.297	75.251
36		79.670	79.623
37		84.167	84.119
38		88.787	88.738
39		93.531	93.480
40		98.399	98.346
41		103.389	101.486
42		108.503	106.505
43		113.741	111.645
44		119.102	116.905
45		124.587	122.287

Table 24. Comparison of doubly excited  ${}_N(K, 1)_n^+ \ ^1\text{P}^0$  ( $N = 4 - 10$ ;  $n = 5 - 14$ ) states of helium ( $Z = 2$ ) with other results. Energies E are in atomic units.

${}_N(K, 1)_n^+ \ ^1\text{P}^0$	Present results	Sakho (2013)	HSC [2]	Ho (1987, 1992, 1996)	Sakho (2010)	Experiment PDA
${}_4^4(2, 1)_5^+ \ ^1\text{P}^0$	0.160 86	0.162 00	0.161 98	0.161 25 <sup>a</sup>	0.162 00	0.162 00
${}_4^4(2, 1)_6^+ \ ^1\text{P}^0$	0.148 78	0.148 90	0.147 85	0.148 05 <sup>a</sup>	0.149 01	0.148 90
${}_4^4(2, 1)_7^+ \ ^1\text{P}^0$	0.141 93	0.141 00	0.140 36	0.140 86 <sup>a</sup>	0.141 79	0.143 10
${}_4^4(2, 1)_8^+ \ ^1\text{P}^0$	0.137 67	0.135 87	0.136 06	0.136 54 <sup>a</sup>	0.137 37	0.139 70
${}_5^5(3, 1)_6^+ \ ^1\text{P}^0$	0.106 83	0.107 88	0.108 21	0.107 29 <sup>bc</sup>	0.108 32	0.105 70
${}_5^5(3, 1)_7^+ \ ^1\text{P}^0$	0.098 76	0.099 36	0.098 63	0.096 55 <sup>bc</sup>	0.099 42	0.099 10
${}_5^5(3, 1)_8^+ \ ^1\text{P}^0$	0.093 85	0.093 92	0.093 04	0.091 52 <sup>bc</sup>	0.094 09	0.094 30
${}_5^5(3, 1)_9^+ \ ^1\text{P}^0$	0.090 65	0.090 22	0.089 64	0.088 67 <sup>bc</sup>	0.090 67	
${}_5^5(3, 1)_{10}^+ \ ^1\text{P}^0$	0.088 44	0.087 58	0.087 41		0.088 35	
${}_5^5(3, 1)_{11}^+ \ ^1\text{P}^0$	0.086 86	0.085 63	0.085 88		0.086 70	
${}_5^5(3, 1)_{12}^+ \ ^1\text{P}^0$	0.085 68	0.084 15	0.084 78		0.085 50	
${}_5^5(3, 1)_{13}^+ \ ^1\text{P}^0$	0.084 78	0.083 00	0.083 96		0.084 59	
${}_6^6(4, 1)_7^+ \ ^1\text{P}^0$	0.076 24	0.076 55	0.076 91	0.076 65 <sup>bc</sup>	0.077 80	0.077 80
${}_6^6(4, 1)_8^+ \ ^1\text{P}^0$	0.070 65	0.070 99	0.070 12	0.070 76 <sup>bc</sup>	0.071 49	
${}_6^6(4, 1)_9^+ \ ^1\text{P}^0$	0.067 05	0.067 21	0.066 69		0.067 49	
${}_6^6(4, 1)_{10}^+ \ ^1\text{P}^0$	0.064 60	0.064 53	0.063 98		0.064 81	

$n(K, 1)_n^+ \text{ } ^1\text{P}^o$	Present results	Sakho (2013)	HSC [2]	Ho (1987, 1992, 1996)	Sakho (2010)	Experiment PDA
${}_6(4,1)_4^+ \text{ } ^1\text{P}^o$	0.062 85	0.062 54	0.062 15		0.062 93	
${}_6(4,1)_5^+ \text{ } ^1\text{P}^o$	0.061 57	0.061 04	0.060 85		0.061 56	
${}_6(4,1)_6^+ \text{ } ^1\text{P}^o$	0.060 59	0.059 86	0.059 90		0.060 54	
${}_7(5,1)_8^+ \text{ } ^1\text{P}^o$	0.057 19	0.057 08	0.058 44		0.058 70	
${}_7(5,1)_9^+ \text{ } ^1\text{P}^o$	0.053 18	0.053 28	0.052 76		0.054 08	
${}_7(5,1)_{10}^+ \text{ } ^1\text{P}^o$	0.050 47	0.050 57	0.049 82		0.051 01	
${}_7(5,1)_{11}^+ \text{ } ^1\text{P}^o$	0.048 57	0.048 57	0.047 79		0.048 88	
${}_7(5,1)_{12}^+ \text{ } ^1\text{P}^o$	0.047 17	0.047 06	0.046 31		0.047 35	
${}_7(5,1)_{13}^+ \text{ } ^1\text{P}^o$	0.046 12	0.045 87	0.045 23		0.046 21	
${}_8(6,1)_5^+ \text{ } ^1\text{P}^o$	0.044 52	0.044 19	0.046 26		0.045 91	
${}_8(6,1)_6^+ \text{ } ^1\text{P}^o$	0.041 55	0.041 47	0.042 16		0.042 44	
${}_8(6,1)_{11}^+ \text{ } ^1\text{P}^o$	0.039 47	0.039 47	0.039 53		0.040 04	
${}_8(6,1)_{12}^+ \text{ } ^1\text{P}^o$	0.037 96	0.037 95	0.037 75		0.038 33	
${}_8(6,1)_{13}^+ \text{ } ^1\text{P}^o$	0.036 83	0.036 77	0.036 49		0.037 06	
${}_9(7,1)_9^+ \text{ } ^1\text{P}^o$	0.035 65	0.035 21	0.036 45		0.036 92	
${}_9(7,1)_{11}^+ \text{ } ^1\text{P}^o$	0.033 39	0.033 22	0.033 40		0.034 25	
${}_9(7,1)_{12}^+ \text{ } ^1\text{P}^o$	0.031 76	0.031 70			0.032 34	
${}_9(7,1)_{13}^+ \text{ } ^1\text{P}^o$	0.030 55	0.030 51			0.030 95	
${}_9(7,1)_{14}^+ \text{ } ^1\text{P}^o$	0.029 62	0.029 58			0.029 89	
${}_{10}(8,1)_4^+ \text{ } ^1\text{P}^o$	0.029 19	0.028 72			0.030 34	
${}_{10}(8,1)_{12}^+ \text{ } ^1\text{P}^o$	0.027 44	0.027 21			0.028 25	
${}_{10}(8,1)_{13}^+ \text{ } ^1\text{P}^o$	0.026 14	0.026 03			0.026 71	
${}_{10}(8,1)_{14}^+ \text{ } ^1\text{P}^o$	0.025 15	0.025 09			0.025 56	

**Table 25.** Comparison of excitation energy of  ${}_3(1,1)_4^+ \text{ } ^1\text{P}^o$  of He-like ( $Z = 2–10$ ). The energies are calculated with respect to the ground state of the corresponding system. The ground state energies taken from Pekeris [32, 33] are the following (in eV): He I: -79.01; Li II: -198.09; Be III: -371.59; B IV: -599.49; CV: -881.82; N VI: -1218.56; OVII: -1609.40; F VIII: -2055.32; Ne IX: -2555.33. The infinite Rydberg energy ( $1 \text{ Ryd} = 13.60569 \text{ eV}$ ) is used for energy conversion. The uncertainties in the experimental measurements are given into parenthesis.

${}_3(1,1)_4^+ \text{ } ^1\text{P}^o$						
Z	Present results	Sakho (2017)	Tang <i>et al.</i> (1992)	Scully <i>et al.</i> (2006) (R-matrix)	Scully <i>et al.</i> (2006) (ALS)	
2	71.63	71.61	71.61			
3	180.16	180.16		180.19		180.23 (5)
4	338.40	338.36				
5	546.31	546.26				
6	803.92	803.89				
7	1111.22	1111.18				
8	1467.90	1467.85				
9	1874.93	1874.88				
10	2331.33	2331.24				

**Table 26.** Comparison of radial expectation value  $\langle r_{12}^{-1} \rangle$  of the  ${}_3(1,1)_n^A \text{ } ^{1,3}P^o$   $n = 4 – 10$  autoionizing states of He-like systems ( $Z = 2–10$ ). The results are expressed in atomic units.

${}_3(1,1)_n^+ \text{ } ^1\text{P}^o$								
n	4	5	6	7	8	9	10	
Z	Present results	Sakho (2017)	Present results					
2	0.076	0.075	0.049	0.035	0.026	0.020	0.016	0.013
3	0.122	0.122	0.080	0.056	0.042	0.032	0.025	0.021
4	0.169	0.168	0.110	0.078	0.057	0.044	0.035	0.029
5	0.216	0.214	0.141	0.099	0.073	0.056	0.045	0.036
6	0.262	0.261	0.171	0.121	0.089	0.069	0.055	0.044
7	0.309	0.307	0.202	0.142	0.105	0.081	0.064	0.052
8	0.356	0.353	0.232	0.163	0.121	0.093	0.074	0.060
9	0.402	0.400	0.263	0.185	0.137	0.106	0.084	0.068
10	0.449	0.446	0.293	0.206	0.153	0.118	0.093	0.076

${}_3(1,1)_n^- \text{ } ^3\text{P}^0$								
n	4	5	6	7	8	9	10	
Z	Present results	Sakho (2017)	Present results					
2	0.068	0.066	0.045	0.033	0.024	0.019	0.015	0.012
3	0.108	0.106	0.073	0.052	0.039	0.030	0.024	0.020
4	0.148	0.145	0.100	0.072	0.054	0.042	0.034	0.027
5	0.188	0.185	0.128	0.092	0.069	0.054	0.043	0.035
6	0.229	0.224	0.155	0.112	0.084	0.065	0.052	0.043
7	0.269	0.263	0.183	0.131	0.099	0.077	0.061	0.050
8	0.309	0.303	0.210	0.151	0.114	0.088	0.071	0.058
9	0.349	0.342	0.238	0.171	0.128	0.100	0.080	0.065
10	0.390	0.382	0.265	0.191	0.143	0.111	0.089	0.073

## 4. Conclusion

In this work, the variationnal procedure of the Screening constant by unit nuclear charge (SCUNC) method has been applied for the first time to the calculations of resonance energies and excitation energies of the doubly  ${}^A_N(K, T)_n^{1,3}P^0$  excited states of the He-like systems. A new correlated wave function adapted to the correct description of the electron-electron correlations phenomena in the doubly excited  $Nlnl'$  states of the He-like systems has been constructed. The good results obtained in this work indicate the possibility to apply the variationnal procedure of the SCUNC method to the treatment of atomic spectra in two electron systems and probably in more complex atomic systems. Study is in such direction.

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