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Leptonic Pair Production in Electro Magnetic Field

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Abstract: At very high energies, the pair production formation through (Photon with Nucleus produce Leptons and Anti-Leptons) exhibits a variety of intriguing properties. Our present objective was to study the electromagnetic fields (DCS) of sulfur nuclei and their effects on high energy lepton pair production. Analytically and quantitatively, the formation of Leptonic pairs in the Electromagnetic field of light (sulfur) nuclei was calculated by the Beth-Hitler equation for the leptonic pair production process. In Ultra-Relativistic (UR) areas of incident photon energy, applying the resulting formulas to the energy distribution of the leptonic pair production process. When we compare the results, we can observe that the Magnetic field of the target nucleus is more efficacious than the Electric field of the nucleus in the leptonic pair production process. Furthermore, we can show that in Pair Production process, the Differential Cross Section (DCS) owing to the target nucleus's Electric Quadrupole (EQ) and Magnetic Octupole (MO) are bigger than the Differential Cross Section (DCS) attributable to the target nucleus's Electric Charge (EC) distribution and Magnetic Dipole (MD). A lighter-mass nucleus is more effective than a higher-mass nucleus. From this, we conclude that the lower the mass number, the better the production of the pair.

Keywords: Pair Production, Positron, Differential Cross Section, The Bethe-Hitler Equation

1. Introduction

There are numerous particles in nature, each of which is accompanied by its field as it moves. They are split into Pheromones and Bosons. Pheromones are divided into Leptons and quarks. The Electron, Muon, and Tau are three charged Leptons, and neutrinos are three neutral Leptons [1]. Both leptons will be studied. The electron is the first lepton, discovered in 1897 by J. Thomson and remains the typical elementary particle. Anderson studied Pair Production for the first time in 1932 when he exploited the process to find the Positron [2]. Bethe and Heitler's work shaped our present theoretical understanding of Pair Production [3]. Muons were discovered by Carl D. Anderson and Seth Neddermeyer in 1936 while examining cosmic radiation [4]. Martin Louis Perle found Tau in a series of experiments between 1974 and 1977 [5].

Nishina and others [6], Bethe and Heitler [7] were the first to theorize the theoretical treatment of (e^-, e^+) Pair production in 1934. In 1936, Jaeget and Hulme [8] established that Pair Production Differential Cross Section (DCS) calculations produce better outcomes at high incident photon energy. Hubbell [9] provides a historical overview of the (e^-, e^+) by photons from Dirac's prediction of the position in 1928 until 2006. The (DCS) results for (e^-, e^+)-Hubell and Seltzer [10] revealed photon-based Pair Production. There is still much scientific research on this topic, and in 2020, Sadah has studied the effect of nuclear magnetic distribution on the photon production of longitudinally polarized lepton-pairs in the field of Na_{11}^{23} and Al_{13}^{27} nuclei [11].

The formation of Leptons is studied in this work through high-energy collisions on the Sulfur nucleus, where the Atomic Number and the Mass Number are 16,32 respectively, we use are generated using electromagnetic fields through high energies of incident photons (from 1 to 5 GeV). The results are given in figures to show the difference in the energy distribution of (l^-, l^+)-Pairs. The obtained results are discussed in detail.

2. Formulation of the Problem

We are studying the effect of high energies on a light nucleus in the Pair Production process.

3. Research Objectives

This study presents new ideas for developing electromagnetic processss and their various applications. This is done by studying the Electro-Magnetic Fields (DCS) of sulfur nuclei and their effects on leptonic pair production at high energy.

4. Research Methodology

The interaction of a photon with the nucleus of an atom produces Pairs of Leptons and Anti-Leptons. The (l^-, l^+) process produced in the interaction of the γ- photon field with the field of nuclei (N) can be written [9, 13]:

$$\gamma(k) + N(Ze) \longrightarrow l^-(p_-) + l^+(-p_+)$$

And the scientist Feynman represented this process with Feynman's drawings, and then this process was studied by the two scientists, Bethe and Hitler, and likewise with equations as in the reference [3, 11, 15].

We can write the Bethe-Hitler equation for the (l^-, l^+)Pair Production process as follows:

$$d(\mathrm{E}, Ze, \mu_1, Q, \Omega) = d1(\mathrm{E}) + d2(\mathrm{E}) + d11(\mathrm{E}) + d22(\mathrm{E}) \tag{1}$$

where

$$d1(\mathrm{E}) = 8\pi\eta \left[\frac{1-\gamma}{4k^2\omega^2\Delta_0} - \frac{1-\gamma^2}{8\beta^2\Delta_0{}^2} + \frac{2\gamma-7}{8k^2\omega^2\Delta_0} - \frac{\varepsilon_0}{4k^2\omega^2\Delta_0(1-\gamma)} - \frac{\varepsilon_T}{8k^2\omega^2\Delta_0(1-\gamma)} * \left[3\gamma + \frac{k^2\omega^2}{\beta^2}\gamma(1-\gamma)^2 \right] + \frac{L}{8k^2\omega^2\Delta_0{}^2(1-\gamma)}[2 + 2(1-\gamma)^2 + \gamma(2-\gamma)\Delta_0] \right] d\Omega \tag{2}$$

$$d2(\mathrm{E}) = 8\pi\eta \left(\frac{\mu_1}{Ze}\right)^2 a_\mu \left[\frac{1}{2}\left[-3 + \frac{k^2\omega^2}{\beta^2\Delta_0}\gamma(1-\gamma-\Delta_0) + \frac{\varepsilon_0}{(1-\gamma)}\left\{ 1-\gamma + \frac{1+(1-\gamma)^2}{\Delta_0} \right\} + \frac{\varepsilon_T}{\beta^3}k^2\omega^2\frac{\gamma^2(1-\gamma+\gamma\Delta_0)}{(1-\gamma)} + \frac{L}{2(1-\gamma)} \right] \right] d\Omega \tag{3}$$

$$d11(\mathrm{E}) = 8\pi\eta \left(\frac{Q}{Ze}\right)^2 a_q \left[k^2\omega^2(4-\gamma)\Delta_0 - 2k^2\omega^2[1+(1-\gamma)^2] + \frac{k^2\omega^2}{\Delta_0}(1-\gamma)\left[1+(1-\gamma)^2\right] + \frac{\varepsilon_0}{2(1-\gamma)}k^2\omega^2[1+(1-\gamma)^2](2-\Delta_0) \right] d\Omega \tag{4}$$

$$d22(\mathrm{E}) = 8\pi\eta \left(\frac{\Omega}{Ze}\right)^2 a_\Omega \left[k^4\omega^4 \left[\frac{20}{3\Delta_0}(1-\gamma)^3\{1+(1-\gamma)^2\} - 12(1-\gamma)^4 + 2(1-\gamma)\{5+2\gamma+7(1-\gamma)^2\} - 4\Delta_0(2-\gamma^2+\gamma^3) - \frac{4}{3}\{6+(1-\gamma)^2\}\Delta_0 + \frac{\varepsilon_0}{(1-\gamma)}\{1+(1-\gamma)^2\}\Delta_0(2+\Delta_0) \right] \right] d\Omega \tag{5}$$

Ze, μ_I, Q, Ω are the (EC), the (MD), the (EQ), and the (MO) moments of the target nucleus [3, 11, 15].
Where:

$$\beta_0 = \sqrt{(1-\gamma)^2 + 2\gamma\Delta}, \beta = k\omega\beta_0, L = 2\ln\left[\frac{2\omega(1-\gamma)}{\gamma}\right], \gamma = \frac{\omega}{E} = \frac{\varepsilon_\gamma}{E} \tag{6}$$

$$\varepsilon_0 = 2\ln[2\omega(1-\gamma)], \varepsilon_t = \ln\left[\frac{\beta-\gamma+1}{\beta+\gamma-1}\right] \tag{7}$$

$$a_\mu = \frac{s+1}{3s} \tag{8}$$

$$a_q = \frac{1}{180}\frac{(s+1)(2s+3)}{s(2s-1)} \tag{9}$$

$$a_\Omega = \frac{2}{4725}\frac{(s+1)(s+2)(2s+3)}{s(s-1)(2s-1)} \tag{10}$$

are the (MD), (EQ), and the (MO) coefficients of the nucleus with spin s.

5. Results and Discussion

The (EC) $d1$, (MD) $d2$, (EQ) $d11$, (MO) $d22$, total Electric dE, and total Magnetic dM Differential Cross Section (DCS) for the (l^-, l^+) using formulas for the energy distribution are obtained for the nucleus S_{16}^{32} and for different values of incident photon energies

$$\varepsilon_\gamma = (1\ Gev, 3\ Gev, 5\ Gev),$$

where $m_e = 9.109558 \times 10^{-28}$, $m_\mu = 1.900318433970471 \times 10^{-25}$, $m_\tau = 0.31677916108494625 \times 10^{-23}$.
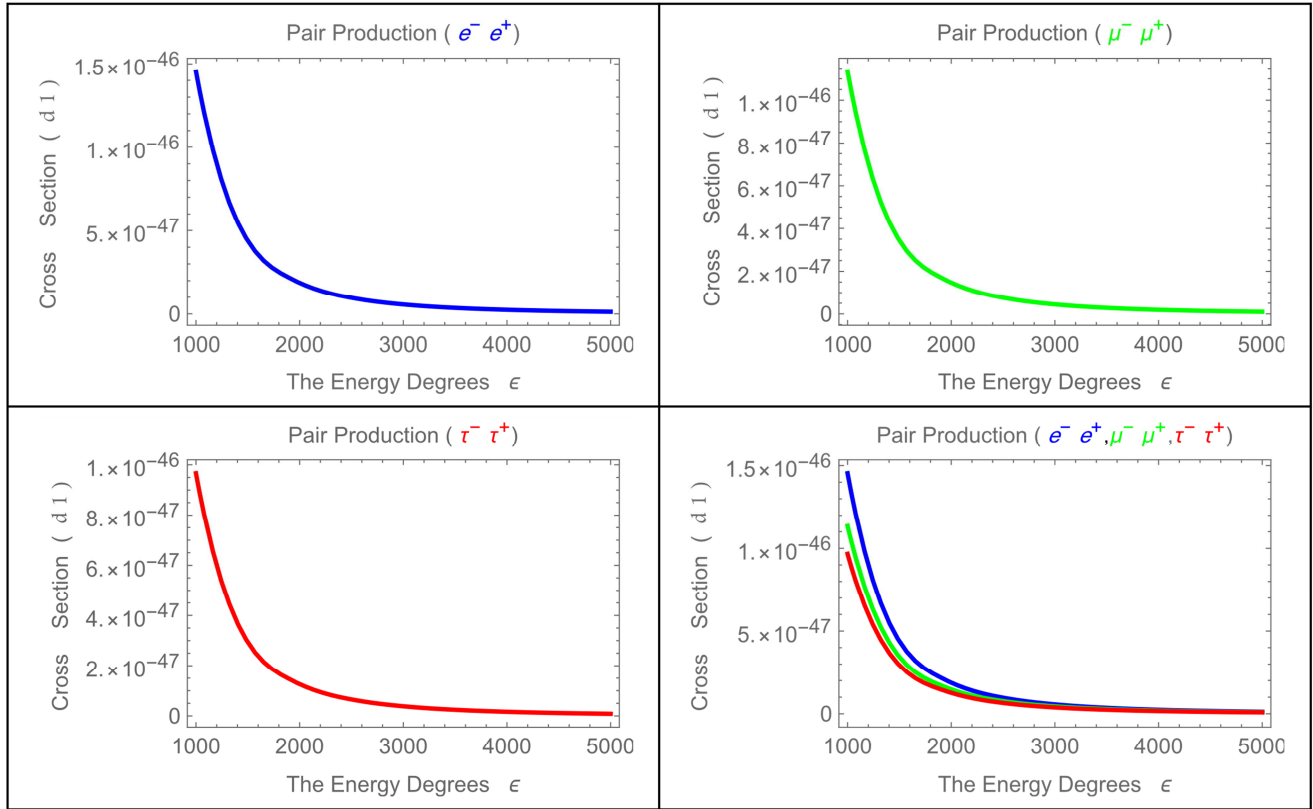
We have summarized the result in the Figures 1-4:

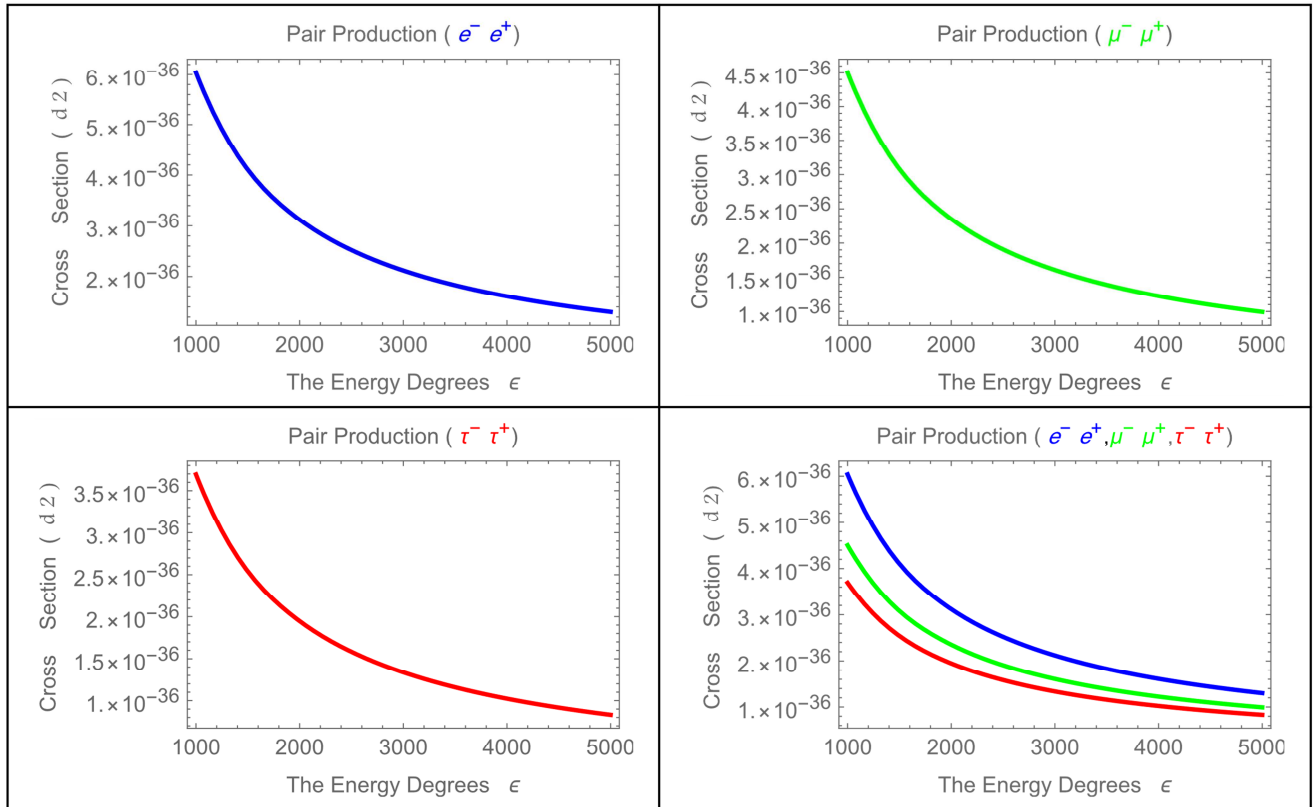

Figure 1. *Electric Charge d1 for the* (l^-, l^+).
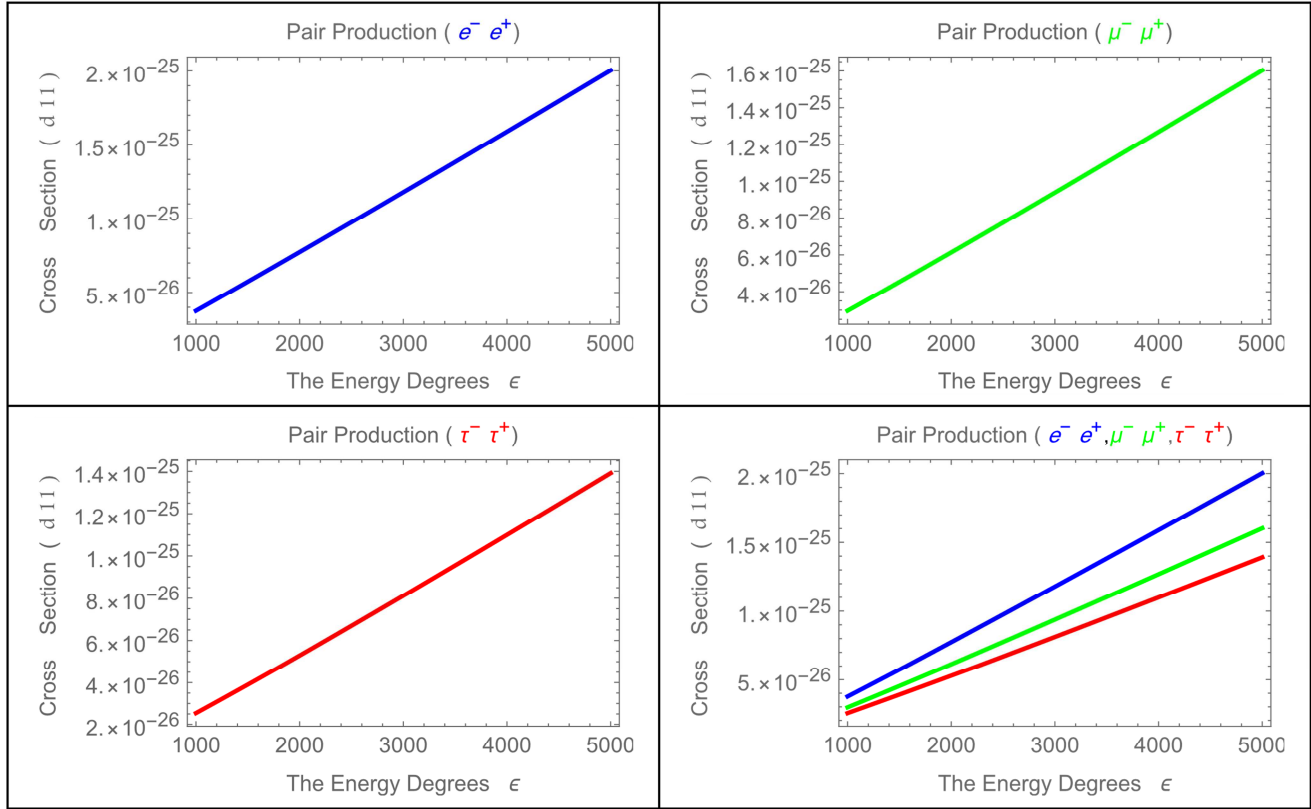

Figure 2. *Magnetic Dipole d2 for the* (l^-, l^+).

Figure 3. *Electric Quadrupole d11 for the* (l^-, l^+).
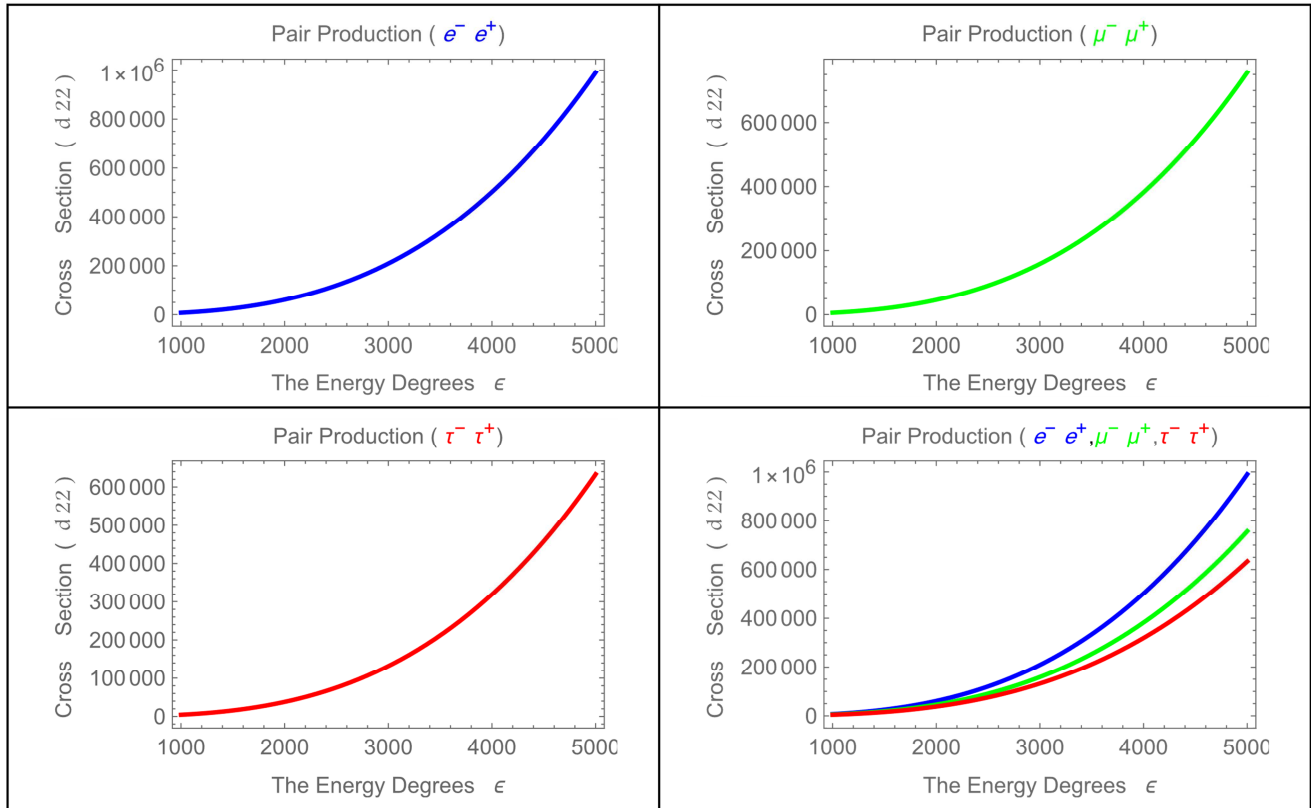

Figure 4. *Magnetic Octupole d22 for the* (l^-, l^+).

From Figures 1-4 we conclude that:

(1) In Figure 1 note the (DCS) $d1$ for (S_{16}^{32}) nucleus are

decreases with increasing energies for the (l^-, l^+).

(2) In Figure 2 note also the (DCS) $d2$ for (S_{16}^{32}) nucleus are decreases with increasing energies for the (l^-, l^+).

(3) As for Figure 3 note the (DCS) $d11$ for (S_{16}^{32}) nucleus are increases with increasing energies for the (l^-, l^+).

(4) In Figure 4 note also the (DCS) $d22$ for (S_{16}^{32}) nucleus are increases with increasing energies for the (l^-, l^+).

(5) The (DCS) of the (l^-, l^+), (EQ) $d11$, and (MO) $d22$ is larger than the (DCS) of (EC) $d1$ and (MD) $d2$ of the target nucleus.

For the Total Electric $dE = d1 + d11$ and the Total Magnetic $dM = d2 + d22$ (DCS) we have also the following result:

(1) The (DCS) $d11$ is more efficacious than the (DCS) $d1$ for the (DCS) dE, i.e., $dE \approx d11$.

(2) The (DCS) $d22$ is more efficacious than the (DCS) $d2$ for the (DCS) dM, i.e., $dM \approx d22$.

(3) Magnetic cross-sections produce more leptonic pairs (l^-, l^+) than electrical cross-sections.

(4) The value of Magnetic cross-section dM for the (e^-, e^+) -pair are larger than that for the pair (μ^-, μ^+) and pair (τ^-, τ^+).

Comparing the values of (DCS) (MO) d22 in Figure 4 with that (EQ) d11 in Figure 3 for (S_{16}^{32}) nucleus, we can show the important result that the (MO) is more efficacious than the (EQ) in the Lepton Pair's production, which means the increasing of the probability of the production of the lepton pairs, confirming the results obtained in a previous work [12] for (B_5^{11}) nucleus and [15] (N_7^{14}) nucleus.

The results were presented in the previous diagrams, which show the energy distribution curves for different leptonic pairs $e^- e^+$, $\mu^- \mu^+$, $\tau^- \tau^+$ and for different values of energies ($\varepsilon_\gamma = 1\ Gev$ from up to $\varepsilon_\gamma = 5\ Gev$) for (S_{16}^{32}) nucleus. From Figures 1-4 obtained for (S_{16}^{32}) nucleus, we see that the production of (e^-, e^+)- pair is larger than that for the (μ^-, μ^+) or (τ^-, τ^+) -pairs. As for the (B_5^{11}) nucleus in [12], we notice that the production of (τ^-, τ^+) pair is larger than that for the (e^-, e^+) or (μ^-, μ^+) -pairs.

6. Conclusion

Compared to reference [15], we found that the process of producing an (l^-, l^+) Pair in a Nitrogen nucleus of lighter mass where the Mass Number is 14 is more effective than in a Sulfur nucleus of higher mass where the Mass Number is 32. We conclude from this that the lower the mass number, the better the production of the pair. Moreover, we can see that in Pair Productions (l^-, l^+), the Magnetic field of the target nucleus is more efficacious than the Electric field of the nucleus. The effect of the (QE) and (OM) (DCS) is more influential in the Lepton Pair's production than the (EC) and (MD). The value of Magnetic cross-section at (e^-, e^+) -pair is better than that for (μ^-, μ^+) and (τ^-, τ^+) pairs.

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